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Equity Misvaluation and Default Options

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ABSTRACT

We study whether default options are mispriced in equity values by employing a structural equity valuation model that explicitly takes into account the value of the option to default (or abandon the firm) and uses firm-specific inputs. We implement our model on the entire cross section of stocks and identify both over- and underpriced equities. An investment strategy that buys undervalued stocks and shorts overvalued stocks generates an annual four-factor alpha of about 11% for U.S. stocks. The model's performance is stronger for stocks with a higher value of the default option, such as distressed or highly volatile stocks.

A LARGE FINANCE LITERATURE ARGUES that equity securities are subject to potential misvaluation by investors (see Harvey, Liu, and Zhu (2016) for a recent survey), and that the extent of misvaluation impacts corporate decisions such as merger and acquisition activities, stock issuance and repurchases, and investment policy. The sources of equity misvaluation are attributed primarily to cognitive biases (see, e.g., Daniel, Hirshleifer, and Subrahmanyam (1998), Barberis, Shleifer, and Vishny (1998), Hong and Stein (1999), and Baker and Wurgler (2006)).

In this paper, we provide a new direction by arguing that equity misvaluation is driven at least in part by investors' failure to fully recognize and adequately price the optionality of equity. It has long been recognized in the finance literature that the equity of a firm with debt in its capital structure is analogous to a call option written on the assets of the firm (see, e.g., Merton (1974)). The title of the seminal paper by Black and Scholes (1973) reflects the applicability of their model to the valuation of corporate debt and equity. Today, nearly every corporate finance textbook (see, e.g., Brealey, Myers, and Allen (2016)) discusses the option-based approach to valuing equity and debt.

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Our paper addresses the question of whether analysts and investors incorporate this option-based approach in their equity valuations, and whether their failure to do so gives rise to misvaluation.

While the option to default is a key characteristic of equity, standard stock valuation techniques such as multiples valuation or discounted cash flow do not explicitly account for this option. Using these techniques therefore can lead to under- or overvaluation, especially among stocks with a relatively high default option value (stocks with greater prospects of default or exit). We build a structural equity valuation model that explicitly accounts for the value of the option to default (or abandon the firm). We then use our option-based valuation model to identify potential misvaluation of equity by examining whether our model can predict future stock returns, and whether its predictive ability is driven by the mispricing of default options.

Our model shares features with other structural valuation models of debt and equity such as endogenous default (see Leland (1994) and a number of models that followed).[1] It also implicitly accommodates the path-dependence of Brockman and Turtle (2003) (due to an additional financial distress cost in low cash flow states). Our model further allows for different tranches of debt with different maturities and additional costs of financial distress. Our model does not incorporate some features that have received considerable attention in corporate finance literature such as investments or managerial entrenchment (see Ozdagli (2010) for a more carefully calibrated model of default), as it focuses specifically on valuing the default option. Notably, we use firm-specific accounting inputs when implementing the model and hence are able to generate firm-level valuations. To the best of our knowledge, our paper is the first to employ a structural option pricing model of default on a large cross section of stocks to value equity and measure potential misvaluation at the firm level.

Leaving the details of our model for later, we start our analysis by sorting all stocks each month into decile portfolios according to the ratio of the model value to market value of equity.[2] We find that most misvalued stocks, either over- or undervalued, are smaller, are more volatile, are less liquid, have lower analyst coverage with higher analyst forecast dispersion, and have lower institutional ownership than more fairly valued stocks, indicating that the former stocks are more difficult to value. Excess returns on these stocks show patterns consistent with our valuation model. The monthly excess return for the most overvalued decile of stocks is 0.51%, while that for the most undervalued decile of stocks is 1.15%.

An analysis of factor loadings reveals that our strategy loads strongly on well-known anomalies of size, value, momentum, and profitability in an

[1] For example, Goldstein, Ju, and Leland (2001) develop a model with dynamic refinancing, Hackbarth, Miao, and Morellec (2006) model the effect of time-varying macroeconomic conditions, and Bharma, Keuhn, and Strebulaev (2010) embed a structural model of credit spreads in a consumption-based asset pricing model.

[2] Sorts based on differences between model value and market value are often used in the literature. See, for example, D'Mello and Shroff (2000), Claus and Thomas (2001), Dong et al. (2006), and Pástor, Sinha, and Swaminathan (2008).

interesting way. Overvalued stocks are closer to small, value, past return loser, and unprofitable stocks than are undervalued stocks. Nevertheless, alphas from a variety of factor models are positive and strongly statistically significant. For example, the four-factor alpha for the overvalued decile of stocks is −0.24%, while that for the undervalued decile stocks is 0.67%. The long-short strategy that buys stocks that are classified as undervalued by our model and shorts overvalued stocks thus generates a four-factor annualized alpha of about 11%. These results are stronger for equal-weighted portfolios (four-factor annualized alpha of about 16%). Our results are robust to the horizon effects discussed in Boguth et al. (2016), conditional alphas as in Boguth et al. (2011) and Cederburg and O'Doherty (2016), various subsamples, and longer holding periods, and are also confirmed using Fama and MacBeth (1973) regressions.

Motivated by the patterns in factor loadings, we next explore the role of the default option more directly by investigating how the returns generated by the model vary across stocks with characteristics related to the default option. We focus on four firm characteristics in this exercise: financial distress, size, volatility, and profitability. Specifically, we first sort all stocks into quintiles based on each of these characteristics. We then double-sort all stocks within each quintile into quintiles according to the model-to-market value ratio. The fraction of the default option value in the total model equity value clearly indicates a relation between the importance of the default option and each of the characteristics. Most notably, for the top distress quintile, the option to default accounts on average for 35.9% of equity value, compared to only 19.2% for the least distressed stock quintile (these numbers also reflect the value of the abandonment option, which is always positive in our model as long as fixed costs are nonzero). The difference in the fraction of the default option between the two extreme size quintiles is 11%, between the two extreme volatility quintiles is 15%, and between the two extreme profitability quintiles is 7%. These relations justify the use of these characteristics, and also support the reliability of our model.

The returns generated by the model also exhibit a clear pattern across the four characteristics. For example, within the top distress quintile, undervalued stocks earn a monthly four-factor alpha that is 1.19% higher than that earned by overvalued stocks. The equivalent difference within the bottom distress quintile is only 0.26%. The model's returns are also much higher among highly volatile stocks, with the four-factor alpha of the long-short strategy equal to 1.38% for the most volatile stocks, but only 0.39% for the least volatile stocks. The effect of firm size, however, is much weaker and not always monotonic: the model's four-factor alpha is 0.75% for small firms and 0.43% for large firms. This is consistent with the fairly low difference in the fraction of the default option between small and large stocks. For profitability-sorted portfolios, we see a reduction in the four-factor alpha of the long-short strategy from 1.09% for the least profitable stocks to 0.69% for the most profitable stocks.

The positive effect of firm characteristics, especially distress and volatility, on the model's performance strongly suggests that the option to default is a primary driver of the predictive ability of the model for future stock returns. To verify this inference, we conduct the following test. We recalculate our model's equity values after shutting down the option to default, and we use these values to resort and recalculate the returns within each characteristic quintile. The model's performance is substantially weaker without the default option. In particular, the model's four-factor alpha decreases from 1.19% to 0.28% for the most distressed stocks, and from 1.38% to 0.33% for the most volatile stocks. These alpha reductions thus show that the option to default is hard to estimate and leads to stock mispricing.

While the return-based evidence already demonstrates our model's predictive ability, we further explore the link between the model's performance and mispricing by performing a battery of additional tests. First, we examine returns around earnings announcements. We expect stronger model performance around earnings announcement days (EADs), as new information about firm fundamentals (and its default likelihood) is revealed to the market and leads to mispricing correction. Our results show strong support for this conjecture. For example, the daily return spread is 24 basis points around EADs, versus only 5 basis points on non-EADs.

Second, we explore proxies for information transparency (analyst coverage, institutional ownership, and availability of listed options). The degree of mispricing is likely to be higher for firms with lower information transparency. We therefore expect the model to perform better for stocks with lower information transparency, with this difference in performance particularly high for stocks with high default option values. Our results are consistent with this prediction. For example, the four-factor monthly alpha of the long-short portfolio sorted on relative model value is about 0.23% higher for low versus high analyst coverage stocks for the quintile of the least distressed stocks, while the same difference is 1.37% for the most distressed stocks.

Third, we analyze the time-series variation in mispricing. We use two measures to capture potential changes in misvaluation over time: the NBER recession indicator and the Baker and Wurgler (2006) sentiment index. We find that our long-short strategy performs better in recessions and during times of high sentiment, in line with our conjecture that valuation difficulties are more pronounced during such times. For example, the four-factor alpha of the hedge portfolio is 1.36% in high-sentiment months but only 0.52% in low-sentiment months.

Fourth, we compare a few postformation variables (equity issuance, institutional ownership, insider trading, probability of mergers, and acquisitions) across stocks classified as over- versus undervalued by the model. We find that overvalued firms are more likely to issue equity, have lower institutional ownership, are sold more actively by insiders, and have a lower probability of being acquired than undervalued firms. Moreover, we find that the differences in these variables are much more strongly pronounced for smaller, more distressed, more volatile, and less profitable stocks. This evidence is consistent with our results being driven by the ability of the model to identify the mispricing of default options.

Finally, as an out-of-sample test, we apply our methodology to the cross section of stocks in nine other most highly capitalized developed markets. Sorting stocks on our model-based relative valuation, we find that, on average across the nine countries, the four-factor alpha of a long-short value- (equal-) weighted strategy is 0.79% (1.19%). This alpha is significant for four (eight) of the countries in our sample. Fama and MacBeth's (1973) regressions provide corroborating evidence: controlling for stock characteristics, the coefficient on relative model value is significant for six out of the nine countries.

Our results suggest that value of the option to default is not properly incorporated in the equity prices in the United States or in the majority of other developed countries. It may seem that ignoring the optionality of equity will always lead to undervaluation of equity. While our model shows that the median U.S. firm is undervalued by about 6.9% in our sample, we do not make the claim that investors are unaware of the possibility of default by equity holders. Our conjecture is simply that investors do not value the resulting option properly.[3] In other words, standard valuation techniques, by employing more crude proxies for this option, lead to misvaluation of equity.

An interesting question that our study raises is why do investors not use option-based valuation models to value stocks. One potential explanation is the complexity involved in implementing such models. We conjecture that many investors, especially retail investors, do not possess the necessary skills to implement such a model. This conjecture is consistent with prior evidence. For example, Poteshman and Serbin (2003) document that investors often exercise call options in a clearly irrational manner, which suggests that it is hard for certain types of investors to understand and value options correctly (Poteshman and Serbin find that this is particularly true for retail investors; traders at large investment houses do not exhibit irrational behavior). Benartzi and Thaler (2001) further show that when faced with a portfolio optimization problem, many investors follow naïve and clearly suboptimal strategies, which suggests that investors fail to fully understand more sophisticated models. In addition, Hirshleifer and Teoh (2003) argue that limited attention and information processing power may lead investors to ignore or underweight information that is important for an option-based model to produce unbiased valuations.

Our paper features a real option model of default and is therefore indirectly related to the growing literature that examines how various properties of real options and their time-varying risk characteristics can affect expected stock

[3] Anecdotal evidence suggests that even top equity analysts do not recognize the default-like features of equity. We studied analyst reports on Ford Motor around late 2008 to early 2009. Ford was in deep financial distress at that time and the option to default was in-the-money. Yet, there is no evidence that analysts from top investment banks incorporated that option value in their analysis. For example, Société Générale based its price estimate on the long-term enterprise-value-to-sales ratio, while Deutsche Bank and JP Morgan used the enterprise-value-to-EBITDA ratio. In addition, Deutsche Bank used a discount rate of 20%, and Crédit Suisse used a Discounted Cash Flow (DCF) model with a "big increase" in the discount rate. These different approaches result in very different values. For instance, the JP Morgan target price for Ford in late October was $2.43 per share, while the Crédit Suisse target price was $1.00 per share.

returns in a rational framework.[4] There are two important differences between our approach and this strand of literature. First, we do not assume that stocks are rationally priced at all times, but rather we build a model to gauge potential equity misvaluation. Second, by using firm-specific inputs, our model allows us to produce valuations at the firm level in every month.

Because our paper focuses on the importance of default options in equity valuations, it is also indirectly related to papers that examine the performance of financially distressed stocks. Dichev (1998) and Campbell, Hilscher, and Szilagyi (henceforth CHS, 2008), among others, show that financially distressed stocks earn surprisingly low returns.[5] In this paper, we do not contribute to the resolution of this puzzle. Unlike the above-mentioned papers, we study the entire cross section of equities and do not focus on the subset of financially distressed stocks. We also note that for the distress risk puzzle to be explained from our misvaluation perspective, it must be the case that investors tend to consistently overvalue default options. This kind of persistent overvaluation would be hard to expect ex-ante and we do not see it in our results.[6] We merely argue that most commonly used reduced-form valuation techniques have the potential of misvaluing (under- and overvaluing) equity embedded with a default option. We therefore emphasize that, while our model is designed to value default options explicitly and default options are more valuable in distressed stocks, the model-to-market value ratio that we use to gauge misvaluation is not directly related to distress. For both distressed and solvent stocks alike, this ratio can be greater or less than one, indicating potential under- or overvaluation of those stocks. Thus, we make a distinction between measures of distress used in the literature and the model-to-market ratio that we use to capture potential misvaluation of equity.

We reiterate that the power of our model is in the valuation of the option to default and/or shut down the firm. For stocks far from the default boundary (e.g., stocks with high cash flows, low volatility, and low leverage ratios), normal valuation techniques are still adequate and not much may be gained by using our model for such stocks. This is confirmed by our empirical results. The performance of the long-short strategy based on our valuation model deteriorates when applied to firms with a low default option value. Finally, while our model can also be used to price corporate debt, the objective of our study is to use it to identify equity misvaluation.

[4] For example, Carlson, Fisher, and Giammarino (2004) model the role of growth option exercise in the dynamics of firm betas, Sagi and Seasholes (2007) focus on convexity in firm value arising from a mean-reverting price process, and Garlappi and Yan (2011) examine how deviations from the absolute priority rule (APR) can affect the risk dynamics of financially distressed stocks.

[5] Chava and Purnanandam (2010), Kapadia (2011), and Friewald, Wagner, and Zechner (2014) offer potential explanations.

[6] The fact that we do not see overvaluation of equity of distressed stocks in our analysis suggests that some other aspects of distressed stocks such as skewness (CHS (2008)) or reduced riskiness due to high shareholder advantage (Garlappi, Shu, and Yan (2008), Garlappi and Yan (2011)) must be driving the returns to distressed stocks. See also Eisdorfer, Goyal, and Zhdanov (2018) for an international study on the determinants of distressed stock returns.

The rest of the paper is organized as follows. Section I describes the model that we use for stock valuation. Section II reports descriptive statistics on portfolios sorted according to the ratio of the equity value implied by our valuation model to the actual equity value. Section III details the performance of our model as indicated by portfolio sorts and Fama and MacBeth (1973) regressions. Additional evidence that our model captures mispricing in the market is provided in Section IV. We report out-of-sample tests on other developed markets in Section V. Section VI concludes.

I. Valuation Model

A key characteristic of corporate equity is the default option. One source of difficulty in valuing equity may therefore come from the necessity of using an appropriate model to account for the value of the option to default. Any valuation model that fails to properly value this option is going to produce values that are further away from fundamental value than a model that does account for this option.

Option pricing structural models have been employed in the literature to gauge the probability of default and to value corporate bonds *given* the value of equity.[7] Here, our objective is to deploy an option pricing model to perform valuation of equity. As we explain in detail later in this section, our option pricing-based approach scores over the traditional approach on two fronts. First, we are better able to estimate future cash flows by explicitly accounting for the exercise of the default option by equity holders. Thus, our model accounts for the truncation in cash flows—at very low states of demand when cash flows are sufficiently negative, it is optimal to exercise the default option rather than to continue to operate the firm. This optionality is missed by commonly employed valuation methods. Second, the estimation of time variation in discount rates is typically a difficult task; it becomes even more difficult for firms with high default risk where any small change in firm value can significantly change the risk of equity. The option pricing approach sidesteps this problem by conducting the valuation under a risk-neutral measure.[8]

Of course, the central insight that the equity of a firm with debt in its capital structure is analogous to a call option written on the assets of the firm dates back to Black and Scholes (1973). While nearly every corporate finance textbook discusses the option-based approach to value equity and debt, academic research on the use of these models to perform equity valuation is sparse. Most of these studies perform valuation of specific types of companies, such as Internet or oil companies, in a real options framework (see Moon and Schwartz

[7] See, for example, Eom, Helwege, and Huang (2004) for a study on the relative accuracy of various structural models in pricing corporate bonds.

[8] A sufficient assumption for this approach to work is that there exists a tradeable asset in the economy whose price is perfectly correlated with the stochastic process that drives the dynamics of the cash flows.

(2000) for an example).[9] By contrast, we implement our model on the entire cross section of stocks.

We assume that the cash flows of firm i are driven by a variable x_{it} that reflects stochastic demand for the firm's products. The firm incurs fixed costs and uses debt, and hence has contractual obligations to make coupon and principal payments to its debt holders. We also assume that a firm with negative free cash flow incurs an additional proportional expense η. This extra cost reflects expenses that a financially distressed firm has to incur to maintain a healthy relationship with suppliers, retain its customer base, and address increased agency costs such as the underinvestment problem or the additional costs of raising new funds to cover for the shortfall in cash flows. The free cash flow to equity holders of firm i is then given by

$$CF_{it} = \left[(1-\tau)(x_{it} - I_{it} - F_i) + \tau Dep_{it} - Capex_{it}\right] \times \left[1 + \eta \mathbf{1}_{(1-\tau)(x_{it}-I_{it}-F_i)+\tau Dep_{it}-Capex_{it}<0}\right] - D_{it}, \quad (1)$$

where x_{it} is the state variable of firm i at time t, I_{it} is total interest payments to debtholders due at time t, D_{it} is the principal repayment due at time t,[10] F_i is the total fixed cost that the company incurs per unit of time, τ is the tax rate, τDep_{it} is the tax shield due to depreciation expense, $Capex_{it}$ is the capital expenditures, and $\mathbf{1}_{(.)}$ is an indicator variable. Note that the additional cost η is incurred only when the cash flow (before the repayment of principal) is negative, so a positive sign on η implies a negative effect on cash flows. In Appendix A, we show that the additional cost η comes into effect once x_{it} crosses a threshold x^* given by equation (A2). We further assume that x_{it} follows a geometric Brownian motion under the physical measure with drift parameter $\mu_{i,P}$ and volatility σ_i:

$$\frac{dx_{it}}{x_{it}} = \mu_{i,P}dt + \sigma_i dW_t. \quad (2)$$

When implementing the model, we use the gross margin (defined as sales less cost of goods sold) as a proxy for the state variable x_{it}. This is in contrast to many structural models that typically use either earnings or unlevered firm value as a state variable that drives valuation.[11] While both cash flows and earnings seem to be reasonable candidates, they pose implementation issues. Earnings is an accounting variable that may not be directly related to valuation. Cash flow data, on the other hand, are often missing from quarterly Compustat data, making it difficult to compute cash flow volatility. Furthermore, cash flows are subject to one-time items such as lump-sum investments, and thus the current

[9] One notable exception is Hwang and Sohn (2010), who model abandonment options in the Black and Scholes (1973) framework and use their model to test predictability of returns on a large cross section of companies. However, they do not model the option to default nor do they incorporate debt structure and fixed costs in their analysis (which are key inputs in our model).

[10] When implementing the model, we further assume that the interest expense is incurred once a year, with the first interest payment due in one year.

[11] See Goldstein, Ju, and Leland (2001) for a criticism of the use of unlevered firm value.

value of cash flows may not necessarily be representative of its evolution in the future. Therefore, to smooth out these potential short-term variations in cash flows, we proxy for x_{it} using the gross margin. Hence, our state variable for firm i in year t is defined as

$$x_{it} = Sales_{it} - COGS_{it}, \tag{3}$$

where $Sales_{it}$ is annual sales and $COGS_{it}$ is the cost of goods sold.

There is a lot of short-term variation in capital expenditures and depreciation. To reduce this noise, we compute the average $Capex/Sales$ ratio for the two-digit SIC industry over the last three years, $\overline{CSR}_{t-3,t}$, and use this ratio and current sales for firm i to proxy for firm i's capital expenditures:

$$Capex_{it} = Sales_{it} \times \overline{CSR}_{t-3,t}. \tag{4}$$

We model depreciation similarly as

$$Dep_{it} = Sales_{it} \times \overline{DSR}_{t-3,t}, \tag{5}$$

where $\overline{DSR}_{t-3,t}$ is the average depreciation-to-sales ratio for the two-digit SIC industry over the last three years. We use selling, general, and administrative expenses (Compustat item $XSGA$) as a proxy for the fixed costs F_i.

We assume that firms issue two types of debt, namely, short-term debt and long-term debt (the model can incorporate any arbitrary maturity structure of debt). We use Compustat annual items DLT (long-term debt) and $DLCC$ (debt in current liabilities) as proxies for a company's long- and short-term debt. We further assume that the short-term debt matures in one year, while the long-term debt matures in five years. Since the coupon rate of debt presumably depends on a company's default likelihood, we model the coupon rate on the long-term debt as the sum of the risk-free rate and the actual yield on debt with a corresponding credit rating. We use the average of the T-bill rate and the 10-year T-note rate as the risk-free rate. For credit rating, we first sort firms into quintiles based on the CHS (2008) distress measure (details on these calculations are provided in Appendix B). We then use AAA, BBB, and BBB+2% yields for distress quintiles 1-2, 3-4, and 5, respectively. We assume that the coupon on the long-term debt is paid annually, where the first payment is due in one year.

We further assume that in year 5, after the long-term debt is paid off (if the firm has not defaulted), the firm refinances its debt by issuing new debt with a perpetual coupon to match the industry average leverage ratio. In year 5, equity holders receive the proceeds from the new debt issuance in exchange for a stream of subsequent coupon payments. Indeed, unless the fixed costs are too high, the firm may find it advantageous to issue new debt rather than stay debt-free due to tax shields on interest payments. We resort to the perpetual coupon assumption to get analytical solutions for post-refinancing values of debt and equity. Refinancing makes our model more realistic by capturing the additional tax benefits that otherwise would be forgone by an all-equity firm. In a previous version of the paper, we also tried a model without refinancing and obtained very similar results. Details on the refinancing procedure are provided in Appendix A.

We denote by GM_{it} the gross margin ratio,

$$GM_{it} = \frac{Sales_{it} - COGS_{it}}{Sales_{it}} = \frac{x_{it}}{Sales_{it}}. \tag{6}$$

We assume that the gross margin ratio stays constant in the future, so future sales are proportional to the state variable x_{it}, in other words, $Sales_{is} = x_{is}/GM_{it}$ for $s \geq t$. We further assume that depreciation and Capex remain proportional to sales (and therefore also proportional to x_{is}):

$$Capex_{is} = Sales_{is} \times \overline{CSR}_{t-3,t} = \frac{x_{is} \times \overline{CSR}_{t-3,t}}{GM_{it}},$$

$$Dep_{is} = Sales_{is} \times \overline{DSR}_{t-3,t} = \frac{x_{is} \times \overline{DSR}_{t-3,t}}{GM_{it}}. \tag{7}$$

To model the growth rate of x_{it} under the physical measure, we use the standard approach discussed in many corporate finance textbooks (see, e.g., Brealey, Myers, and Allen (2016)). We model growth in capital expenditures in continuous time because our general setup is in continuous time. We first posit that capital expenditures generate growth. Thus, $Capex_{it}$ invested over a time interval $[t, t + dt]$ results in an expected (under P) increase in operating cash flow of $Capex_{it} R_A dt$ over the next interval dt, where R_A is after-tax return on assets and (instantaneous) operating cash flow, OCF, is defined as the after-tax gross margin plus depreciation tax shield,

$$OCF_{it} dt = \left[(1-\tau)x_{it} + \tau Dep_{it}\right] dt = x_{it}\left(1 - \tau + \tau \frac{\overline{DSR}_{t-3,t}}{GM_{it}}\right) dt. \tag{8}$$

The expected growth rate in operating cash flows (under P) then equals

$$\frac{Capex_{it} R_A dt}{x_{it}\left(1 - \tau + \tau \frac{\overline{DSR}_{t-3,t}}{GM_{it}}\right) dt} = \frac{x_{it} \frac{\overline{CSR}_{t-3,t}}{GM_{it}} R_A}{x_{it}\left(1 - \tau + \tau \frac{\overline{DSR}_{t-3,t}}{GM_{it}}\right)} = \frac{\frac{\overline{CSR}_{t-3,t}}{GM_{it}} R_A}{\left(1 - \tau + \tau \frac{\overline{DSR}_{t-3,t}}{GM_{it}}\right)}. \tag{9}$$

Because operating cash flow in our setup is proportional to x_{it}, it follows that the drift parameter of the process x_{it} under the physical measure is given by

$$\mu_{i,P} = \frac{\mathbb{E}_t^P(dx_{it})}{x_{it} dt} = \frac{\overline{CSR}_{t-3,t} R_A}{(1-\tau)GM_{it} + \tau \overline{DSR}_{t-3,t}}, \tag{10}$$

where $\mathbb{E}_t^P$ is the conditional expectation under the physical measure P at time t.

We note that while the drift of x_{it} under the physical measure is given by $\mu_{i,P} = R_A - DY_{i,P}$, the growth rate under the risk-neutral measure is given by $\mu_{i,Q} = r - DY_{i,Q}$, where DY is the dividend yield and r is the risk-free rate.

Since the dividend yield is the same under both measures, $DY_{i,P} = DY_{i,Q}$, it follows that

$$\mu_{i,Q} = r - R_A + \mu_{i,P}. \tag{11}$$

We use the risk-neutral growth $\mu_{i,Q}$ in the implementation of the model, in which the pricing is done under the risk-neutral measure Q. To measure the return on assets R_A, we calculate the cost of equity using the CAPM.[12] We estimate firms' betas over the past three-year period and then average across all firms in the same two-digit SIC industry that fall in the same distress quintile based on the CHS (2008) measure of financial distress.[13] We model the cost of debt using the firm's credit rating as described above. The return on assets, R_A, is then equal to the weighted average of the cost of equity capital and the cost of debt.

We proxy for σ using the annualized quarterly volatility of sales over the last eight quarters. If quarterly sales values are not available in Compustat, we use the average quarterly volatility of sales of the firms in the same two-digit SIC industry over the last eight-quarter period. We use the volatility of sales as opposed to the volatility of x_{it} in equation (2) because we believe that it better reflects the volatility of the underlying demand-driven stochastic process, which drives valuation in structural models like ours. Using the volatility of x_{it} instead would capture some short-term variations in the costs of goods sold and capital expenditures that are not related to the underlying economic uncertainty and therefore should not affect the value of the option to default (nevertheless, we get similar results using the volatility of $Sales - COGS$). We use 35% for the corporate tax rate, τ, while we set distress costs, η, to 15%.[14] The model inputs are summarized in Table I.

Default is endogenous in our model, similar to the majority of structural models (see, e.g., Leland (1994)). Equity Holders are endowed with an option to default that they exercise optimally; in particular, they default if continuing to operate the firm results in a negative value. In our model, default occurs when cash flow to equity holders is sufficiently negative.[15] Note that the presence of

[12] Note that the risk-neutral option pricing approach of our model is consistent with the use of the CAPM to estimate the cost of capital R_A. For example, if no cash flows are reinvested, then the growth rate under Q is $r - R_A$, which makes put options on such firms more expensive and call options less expensive. This is consistent with call options having higher (positive) betas for such riskier firms and put options having lower (negative) betas. See also Rubinstein (1976) for a proof of the equivalence of the CAPM and the Black-Scholes option pricing framework.

[13] We differentiate between firms in different distress categories because expected returns to claimholders vary depending on the degree of distress.

[14] Weiss (1990) estimates the direct costs of financial distress to be of the order of 3% of firm value, Andrade and Kaplan (1998) provide estimates between 10% and 23%, while Elkamhi, Ericsson, and Parsons (2011) use 16.5% in their analysis. Our results are robust to different values of η.

[15] Note that we implicitly assume that equity holders of a firm with negative free cash flow may continue to inject cash (issue new equity) into the firm (unless they decide to default), but it is costly do so and this cost is reflected in the parameter η. This assumption is common in structural credit risk and capital structure models. Setting η to infinity would result in immediate default as soon as the cash flow to equity holders turns negative.

Table I
Valuation Model Inputs

This table reports the input parameters used in our valuation model for all CRSP/Compustat firms. The categories of parameters include values that are kept constant for all firms and months, firm-month-specific values, and values based on two-digit SIC industry codes and CHS (2008) distress risk quintiles. The sample period is 1983 to 2012.

Input Variable	Value Used in the Model	Mean	Median	*SD*
Coupon rate	AAA, BBB and BBB+2% yields for distress quintiles 1-2, 3-4, and 5, respectively	8.35%	8.04%	2.36%
Distress costs, η	15%			
Corporate tax rate, τ	35%			
Risk-free rate, r	Average of 3-month and 10-year Treasury yields	5.22%	5.25%	2.56%
R_{WACC}	Average industry distress *WACC* in the last three years	9.39%	9.56%	2.49%
Capex-to-*Sales* ratio, *CSR*	Average industry distress *CSR* in the last three years	0.108	0.066	0.122
Depreciation-to-*Sales* ratio, *DSR*	Average industry distress *DSR* in the last three years	0.079	0.048	0.079
Volatility, σ (annualized)	Quarterly volatility of sales	0.396	0.260	0.440
Short-term debt/Total assets	Annual Compustat items *DLC/AT*	0.057	0.020	0.114
Long-term debt/Total assets	Annual Compustat items *DLTT/AT*	0.169	0.110	0.203
Market leverage ratio	(*DLC*+*DLTT*)/(*DLC*+*DLTT* +Equity value)	0.265	0.191	0.258
Fixed costs/Sales	Annual Compustat items *XSGA/Sales*	0.351	0.244	0.466
Gross margin/Sales	Annual Compustat items (*Sales*−*COGS*)/*Sales*	0.253	0.346	0.874

the fixed cost component, F_i, means that equity holders may decide to shut down operations and abandon the firm if the cash flow turns sufficiently negative even when the firm is debt-free. The option to exit is valuable, even for an all-equity firm as long as F_i is positive.

Stockholders maximize the value of equity (we abstract from any potential conflicts of interest between managers and stockholders). The value of equity, V_0, given the initial state variable x_0 is equal to the expected present value of future cash flows under the risk-neutral measure discounted by the risk-free rate r,

$$E_{i0}(x_0) = \sup_{T_{x_d(t)}} \mathbf{E}_{x_0}^{Q} \int_0^{T_{x_d(t)}} e^{-rt} CF_{it} dt, \tag{12}$$

where $x_d(t)$ is the optimal default boundary and $T_{x_d(t)}$ is a first passage time to the boundary $x_d(t)$ of the process x.[16] The default boundary is a function of time because debt has final maturity, and coupon and principal payments are allowed to vary over time. The equity value can be decomposed into the value that would accrue to equity holders if they were forced to operate the firm forever (the discounted cash flow component) and the value of the default (abandonment) option,

$$\text{Default option} = \sup_{T_{x_d(t)}} \mathbf{E}^Q_{x_0} \int_0^{T_{x_d(t)}} e^{-rt} CF_{it} dt - \mathbf{E}^Q_{x_0} \int_0^{\infty} e^{-rt} CF_{it} dt \geq 0. \quad (13)$$

Equation (13) reflects two fundamental differences between our valuation approach and traditional valuation methods. First, we discount cash flows to equity holders only until the stopping time $T_{x_d(t)}$. This stopping time is determined as the outcome of the optimization problem of equity holders and results from the optimal exercise of the option to default. By contrast, the usual valuation methods implicitly assume an infinite discounting horizon and ignore the option to default (i.e., they value only the second term on the right-hand side of equation (13)). Second, we use the risk-free rate and discount payouts to shareholders under the risk-neutral measure, while the standard valuation methods perform discounting under the physical measure. This also distorts valuations because risk and the appropriate discount rate under the physical measure vary significantly as the firm moves in and out of financial distress. In other words, as is well known, one cannot price an option by expectation under the physical measure.

In Internet Appendix Section III,[17] we allow for time variation in the drift of x_{it} by modeling the return on capital R_A as a mean-reverting process. The quality of investment projects available to managers may vary over time, and they might have access to better or worse projects at different times. True return on investment is often hard to measure precisely ex-ante and can lead to ex-post variation in the actual return. Furthermore, managers may have incentives to deliberately invest in negative NPV projects (those that generate return below R_A) due to the overinvestment/free cash flow problem of Jensen (1986). To account for these effects, we assume a regime-shifting process that generates mean reversion in the growth rate. This alternative approach yields results that are qualitatively similar to our base-case assumption of constant growth.

A potential minor issue with our approach is that a small percentage (on average 5%) of companies have negative current gross margin, while we assume that x_{it} is a geometric process and hence always positive. We exclude

[16] We assume that the APR is enforced and equity holders receive zero payoff upon default. Deviations from the APR and nonzero value of equity in default would induce higher default option values and higher probability of default. See Garlappi and Yan (2011) for equity valuation when APR is violated.

[17] The Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

these companies from our main tests. In robustness checks, however, we follow an alternative approach. Since we cannot assume geometric growth for such companies, we assume that x_{it} follows an arithmetic Brownian motion until the moment it reaches the value equal to its annualized standard deviation (of course, before the company defaults), at which point we assume that x_{it} begins to grow geometrically. We obtain similar results using this alternative approach. Finally, we employ a standard binomial numerical algorithm to determine both the optimal default boundary and the value of equity in equation (12). Further numerical details on the implementation of our procedure are provided in Appendix A.

II. Model Descriptives

We perform our valuation on the entire universe of stocks obtained by merging annual and quarterly Compustat data with the return data from CRSP. Each month we sort all stocks into deciles according to the ratio of the equity value implied by our valuation model to the actual equity value. Decile 1 contains the most overvalued stocks, while decile 10 consists of the most undervalued stocks. This valuation sort is similar in spirit to scaling the market price to predict returns (Lewellen (2004)). While the most usual scaling variable is the book value, some studies use model-implied valuation as a scaling variable. For example, Lee, Myers, and Swaminathan (1999) use the ratio of residual income value to market value to predict future returns. Some other papers that use the ratio of model value to market value to detect equity misvaluation are D'Mello and Shroff (2000), Claus and Thomas (2001), Dong et al. (2006), and Pástor, Sinha, and Swaminathan (2008). Our approach, while using a different valuation method, is similar in its use of the sorting variable.

We present the portfolio characteristics in Table II. In addition to size, market-to-book, market beta (calculated using the past 60 months), past six-month return, and the standard deviation of daily stock returns, we also show the percentage of firms reporting negative earnings, number of analysts, the standard deviation of their forecasts, equity issuance, institutional ownership, and two proxies for liquidity, namely, share turnover and Amihud's (2002) illiquidity measure. Accounting and stock return data are from CRSP and Compustat, and all analyst data are from IBES. For each characteristic, we first calculate the cross-sectional mean and median of each portfolio. We then report the time-series averages of these means and medians. We exclude observations in the top and bottom percentiles in calculating the means and medians. We include all common stocks, although our results are robust to the exclusion of financial stocks. The sample period for our study is 1983 to 2012 as coverage of quarterly Compustat data is sparse before this date.

Table II shows that the most misvalued (over- or undervalued) stocks are smaller, are more volatile, are less liquid (especially undervalued stocks), have lower analyst coverage with higher analyst forecast dispersion, and have lower institutional ownership than more fairly valued stocks. While these observations are not especially surprising, as presumably these are the characteristics

Table II

Descriptives of Portfolios Sorted on Relative Model Value

Each month, we sort all stocks into deciles according to the ratio of the equity value implied by our valuation model to actual equity value (decile 1 = most overvalued, decile 10 = most undervalued). The portfolios are value-weighted and held for one month. The table presents descriptive statistics for each portfolio, where, for all variables, observations outside the top and bottom percentiles are excluded. For each variable, we first calculate the cross-sectional mean and median across stocks for each portfolio. We then report the time-series averages of these means/medians. Size is equity value (in millions of dollars). Market-to-book ratio is equity market value divided by equity book value. Market beta is measured by the regression of stock returns on market returns over the past 60 months. Past return is the cumulative return over the past six months. Standard deviation of daily stock returns (reported in percent) is based on market-adjusted returns in the past year. Share turnover is trading volume scaled by total shares outstanding. Amihud illiquidity is the monthly average of daily ratios of the absolute return to dollar trading volume (in millions). Percent of firms with negative earnings is based on net income in the previous calendar year. Number of analysts covering the firm is measured as the number of forecasts appearing in IBES. Standard deviation of analyst forecasts is also calculated from IBES data. Equity issuance (reported in percent) is measured as the difference between the sale and purchase of common and preferred stocks during the year, scaled by equity market value at the beginning of the year. Institutional ownership (reported in percent) is the sum of all shares held by institutions divided by total shares outstanding. The sample period is 1983 to 2012.

		Decile									
		1	2	3	4	5	6	7	8	9	10
Size	Mean	833.4	1,757.0	2,197.9	2,057.0	2,034.1	1,822.5	1,605.7	1,313.5	1,037.9	435.7
	Median	113.6	260.1	384.1	390.0	357.8	308.9	275.4	220.3	156.8	55.4
Market-to-book ratio	Mean	2.69	2.57	2.40	2.16	1.96	1.79	1.64	1.51	1.34	1.04
	Median	2.11	2.24	2.18	1.96	1.76	1.60	1.46	1.32	1.15	0.80
Market beta	Mean	1.31	1.28	1.15	1.04	0.98	0.94	0.93	0.95	0.95	1.01
	Median	1.19	1.17	1.05	0.93	0.88	0.84	0.83	0.86	0.87	0.94
Past return	Mean	15.9	17.7	15.3	12.6	10.3	8.3	6.6	4.4	1.1	−7.2
	Median	6.6	9.0	9.0	7.7	5.9	4.5	2.7	0.2	−3.3	−12.2
Stdev of stock returns	Mean	4.2	3.6	3.2	3.0	2.9	2.9	3.0	3.2	3.7	4.9
	Median	3.7	3.1	2.7	2.5	2.4	2.4	2.5	2.7	3.1	4.2

(*Continued*)

Table II—*Continued*

		Decile									
		1	2	3	4	5	6	7	8	9	10
Share turnover	Mean	0.12	0.13	0.12	0.11	0.10	0.09	0.09	0.09	0.08	0.08
	Median	0.07	0.08	0.08	0.07	0.07	0.06	0.06	0.06	0.06	0.05
Amihud illiquidity	Mean	5.15	3.28	2.54	2.37	2.49	2.73	3.28	4.55	6.95	14.83
	Median	0.21	0.07	0.05	0.06	0.06	0.07	0.11	0.26	0.63	2.19
% of negative earnings	Mean	60.9	33.6	23.0	17.7	15.7	15.1	15.6	17.8	22.0	35.9
	Median	86.0	10.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.5
Number of analysts	Mean	3.54	4.01	4.22	4.11	3.87	3.79	3.63	3.30	3.19	2.60
	Median	2.80	3.27	3.51	3.47	3.25	3.14	3.00	2.71	2.65	2.15
SD of analyst forecasts	Mean	0.06	0.04	0.04	0.04	0.04	0.05	0.06	0.06	0.06	0.08
	Median	0.04	0.03	0.03	0.03	0.03	0.04	0.05	0.04	0.05	0.06
Equity issuance	Mean	5.9	3.1	1.9	1.3	1.0	1.0	1.0	1.0	1.3	3.0
	Median	0.7	0.2	0.2	0.1	0.0	0.1	0.1	0.0	0.0	0.0
Institutional ownership	Mean	31.5	40.7	44.4	44.3	43.0	41.1	40.0	38.6	35.2	29.5
	Median	25.7	38.9	45.6	45.8	43.6	40.9	38.9	37.4	33.1	25.5

of the stocks that are most difficult to value, they do provide a first indication that our model successfully detects stocks whose market values move further away from fundamental values. The results further show that the most overvalued stocks (decile 1) have, unsurprisingly, higher market-to-book ratios than the most undervalued stocks (decile 10), which also explains their higher market beta. Decile 1 stocks further show higher past returns and issue more equity than decile 10 stocks. These equity issuance patterns are consistent with our valuation model under the additional assumption that managers of these firms know the true valuations and time the market in issuing equity.

To show that our valuation measure is tied to default risk, we compute default probabilities based on our model. Our numerical procedure (see Appendix A for details) allows for the determination of the default boundary (defined as the state of the stochastic variable x at which the equity value approaches zero). For each firm in each quarter, we compute this boundary and then perform 10,000 simulations of the process x under the physical measure. We then compute a default probability over a T-year horizon as the percentage of times out of the 10,000 simulation draws that the firm ends up crossing the default boundary from above in the first T years. We compare these default probabilities to actual default probabilities as well as those from CHS (2008). CHS use logit regressions to predict failure probabilities while incorporating a large set of accounting variables. We find that the correlation between one-year-ahead defaults and our model-implied probability is 0.114, while it is 0.226 for the CHS model. The equivalent numbers for three-year-ahead defaults are 0.185 for our model and 0.266 for the CHS model. Since the CHS model is calibrated based on actual defaults, it is not surprising that it does better than our model in predicting future defaults. Nevertheless, our model still produces a relatively decent correlation with actual defaults. We also calculate accuracy ratios that assess a model's ability to predict actual defaults as in Vassalou and Xing (2004). We find accuracy ratios of 50% for our model that compare favorably with the accuracy ratios of 76% for the CHS model.

As a sharper test of our conjecture that the stocks in the extreme deciles are the most misvalued by the market (potentially due to the market's inability to value the default option correctly), we check whether the stocks in these deciles do, in fact, default more often than more fairly valued stocks. To do so, we calculate the fraction of stocks that default based on CRSP's delisting codes associated with poor performance, such as bankruptcy, liquidation, dropping due to bad performance, etc. We find that the average default rate of stocks in deciles 1 and 10 is 6.6%, 11.0%, and 14.6% in the one, two, and three years after portfolio formation, respectively (in comparison, the default rate of stocks in decile 5 is 1.1%, 2.2%, and 3.4%, respectively). These statistics provide further indication that the misvaluation picked up by our model is related to the default option.

Our structural model produces equity values as a nonlinear function of firm characteristics. It is useful to understand which aspects of valuation go into our model that are not accounted for in simpler models. To shed light on this question, we approximate our model value as a linear function of the model inputs. Specifically, we run cross-sectional Fama and MacBeth (1973) regressions of scaled model value on model inputs each month. The model inputs are the same as those reported in Table I. In results reported in Internet Appendix Section II, we find that the coefficients on various model inputs are in line with general economic intuition. For example, increasing debt (both short- and long-term debt), fixed costs, and the coupon rate all have a negative effect on model values, whereas volatility has a strong positive effect. Of course, the true relation between model values and input parameters is inherently nonlinear (as in most option pricing models). Nevertheless, the linearization of the model provides useful insights in understanding how each of the input variables affects the model output on average. Equally important, this exercise helps us understand that many aspects of valuation that go into our model (and are not accounted for in simpler models) may be potentially useful in explaining stock returns.

III. Model Performance

A. *Portfolio Returns of Stocks Sorted on Model Valuation*

We next check the efficacy of our valuation model by calculating returns of the decile portfolios. We form both value- and equal-weighted portfolios. While value-weighting is more common in the literature, Table II shows that more mispriced stocks are smaller on average. It is thus possible that equal-weighting leads to stronger performance of our sorts. Accordingly, we present most of results for value-weighted portfolios but also show equal-weighted portfolios for reference. Table III reports the monthly returns on each portfolio as well as the returns to the hedge portfolio that is long the most undervalued firm portfolio (decile 10) and short the most overvalued firm portfolio (decile 1). In addition to reporting the average return in excess of the risk-free rate, we report the alphas from one-, three-, and four-factor models. The one-factor model is the CAPM model. We use the Fama and French (1993) factors in the three-factor model. These factors are augmented with a momentum factor in the four-factor model. All factor returns are downloaded from Ken French's website. All returns and alphas are in percent per month and numbers in parentheses are the corresponding t-statistics.

Table III shows that returns and factor-model alphas are generally monotonically increasing as one moves from decile 1 (most overvalued stocks) to decile 10 (most undervalued stocks), which supports our model's ability to detect stock mispricing. The hedge portfolio has excess returns of 0.65% per month (t-statistic = 2.10). Factor-model alphas display patterns consistent with the excess returns and characteristics of stocks shown in Table II. For example, since decile 10 stocks are, on average, smaller and have lower market-to-book ratios than decile 1 stocks, the 10 − 1 portfolio has a lower three-factor alpha of 0.49% than CAPM alpha of 0.82%. At the same time, since past returns for decile 10 stocks are lower than those for decile 1 stocks, the four-factor alpha of the long-short portfolio is higher at 0.91% (t-statistic = 3.68). Regardless of

Table III

Returns of Portfolios Sorted on Relative Model Value

Portfolios are sorted as in Table II (decile 1 = most overvalued, decile 10 = most undervalued). The table shows the portfolios' mean excess monthly returns (in excess of the risk-free rate) and alphas from factor models. The CAPM one-factor model uses the market factor. The factors in the three-factor model are the Fama and French (1993) factors. The factors in the four-factor model are the Fama and French (1993) factors augmented with a momentum factor. We also show the equal-weighted (ew) returns/alphas on the long-short 10−1 portfolio in the last column. All returns and alphas are in percent per month and the corresponding t-statistics are in parentheses. The sample period is 1983 to 2012.

	Decile											ew
	1	2	3	4	5	6	7	8	9	10	10−1	10−1
Excess return	0.51	0.39	0.69	0.66	0.56	0.73	0.78	0.83	1.11	1.15	0.65	1.11
	(1.36)	(1.24)	(2.54)	(2.59)	(2.32)	(2.96)	(3.12)	(3.31)	(4.01)	(3.34)	(2.10)	(4.59)
CAPM alpha	−0.31	−0.33	0.08	0.08	0.03	0.20	0.26	0.30	0.55	0.51	0.82	1.16
	(−1.76)	(−2.73)	(0.69)	(0.81)	(0.24)	(1.65)	(1.92)	(2.29)	(3.49)	(2.23)	(2.68)	(4.77)
Three-factor alpha	−0.21	−0.26	0.10	0.02	−0.11	0.06	0.07	0.13	0.33	0.28	0.49	0.90
	(−1.32)	(−2.21)	(0.89)	(0.18)	(−1.14)	(0.56)	(0.66)	(1.17)	(2.52)	(1.37)	(1.76)	(4.14)
Four-factor alpha	−0.24	−0.29	0.08	0.02	−0.07	0.13	0.18	0.26	0.48	0.67	0.91	1.27
	(−1.54)	(−2.43)	(0.67)	(0.16)	(−0.75)	(1.29)	(1.68)	(2.34)	(3.74)	(3.95)	(3.68)	(6.65)

the risk correction, the alphas of 10 − 1 portfolio are economically large and mostly statistically significant.[18] The last column of Table III shows that, as suspected, the hedge portfolio returns on equal-weighted portfolios are even higher: the four-factor alpha is 1.27% (t-statistic = 6.65).

A.1. Alternate Model Alphas

We next calculate alphas from alternative factor models. The factors that we use include FF5, the Fama and French (2015) five-factor model; HXZ, the Hou, Xue, and Zhang (2015) q-factor model; MOM, the momentum factor based on past returns over the last 12 months skipping the most recent month (Jegadeesh and Titman (1993)); REV, the reversal is based on past month returns (Jegadeesh (1990)); BAB, the Frazzini and Pedersen (2014) betting-against-beta factor; and LIQ, the Pástor and Stambaugh (2003) liquidity factor. We use the REV factor based on concerns that our holding period is only one month, and the LIQ factor based on concerns that the stocks in the extreme deciles are generally illiquid. We use various combinations of these factors and report the results in Table IV.

Alphas for the 10 − 1 portfolio are significant in all factor models that we consider. The magnitudes are obviously different but, even so, the lowest value- (equal-) weighted portfolio alpha that we find is 0.47% (1.12%). The most comprehensive factor models that we use (those that include the MOM, REV, and LIQ factors) give alphas of 0.56% (t-statistic = 2.24) with FF5 as standard factors and 0.84% (t-statistic = 2.88) with HXZ as standard factors.

Finally, we check whether the portfolio returns can be explained by a volatility factor. We find that the loading of the 10 − 1 portfolio return on changes in VIX (a proxy for the volatility factor) is economically small at −0.009 with a statistically insignificant t-statistic of −0.11.

A.2. Factor Model Loadings

We also check for factor model loadings. Recall that we note above that decile 10 stocks are, on average, smaller and have lower market-to-book ratios than decile 1 stocks. Panel A of Table V shows the loadings of decile portfolios on the four-factor model that includes the three Fama and French (1993) factors and the momentum factor. We find that decile 1 stocks (most overvalued stocks) are similar to growth stocks, while decile 10 stocks (most undervalued stocks) are similar to small, value, and past-return loser stocks. Consequently, the 10 − 1

[18] Some readers have suggested that postformation returns are not necessarily a sufficient test of our valuation model, especially if market valuation drifts even further away from our "fair" valuation. We check this by computing the value gap, that is, the difference between market valuation and our valuation. We calculate the value gap at portfolio formation and one quarter after portfolio formation. We verify that the value gap does indeed shrink on average one quarter after portfolio formation. At the same time, the value gap does not decline to zero, suggesting that the correction takes longer than one quarter (see also the robustness checks on long-horizon returns in Internet Appendix Section IA I.D).

Table IV

Different Factor Model Alphas of Portfolios Sorted on Relative Model Value

Portfolios are sorted as in Table II (decile 1 = most overvalued, decile 10 = most undervalued). The table shows the portfolios' alphas from factor models. FF5 is the Fama and French (2015) five-factor model. HXZ is the Hou, Xue, and Zhang (2015) q-factor model. MOM is the momentum factor based on past returns over the past 12 months skipping the most recent month (Jegadeesh and Titman (1993)). REV is the reversal factor based on past month returns (Jegadeesh (1990)). BAB is the Frazzini and Pedersen (2014) betting against beta factor. LIQ is the Pástor and Stambaugh (2003) liquidity factor. We also show the equal-weighted (ew) returns/alphas on the long-short 10 − 1 portfolio in the last column. All returns and alphas are in percent per month and the corresponding t-statistics are in parentheses. The sample period is 1983 to 2012.

	Decile											ew
	1	2	3	4	5	6	7	8	9	10	10−1	10−1
FF5 + MOM	0.11	−0.19	−0.05	−0.16	−0.25	−0.05	0.00	0.11	0.39	0.63	0.52	1.15
	(0.73)	(−1.57)	(−0.48)	(−1.72)	(−2.76)	(−0.50)	(0.04)	(1.01)	(2.99)	(3.61)	(2.22)	(6.05)
FF5 + MOM	0.13	−0.20	−0.06	−0.18	−0.27	−0.06	−0.02	0.10	0.37	0.60	0.47	1.12
+ BAB	(0.88)	(−1.66)	(−0.52)	(−2.08)	(−2.99)	(−0.62)	(−0.16)	(0.88)	(2.87)	(3.46)	(2.03)	(5.92)
FF5 + MOM	−0.07	−0.18	−0.06	−0.10	−0.21	0.04	0.06	0.07	0.50	0.49	0.56	1.20
+ REV +LIQ	(−0.46)	(−1.36)	(−0.52)	(−1.07)	(−2.14)	(0.38)	(0.55)	(0.59)	(3.58)	(2.67)	(2.24)	(5.92)
HXZ + MOM	0.05	−0.25	−0.09	−0.19	−0.28	−0.07	−0.03	0.11	0.43	0.79	0.74	1.21
	(0.31)	(−2.04)	(−0.75)	(−2.11)	(−2.96)	(−0.64)	(−0.26)	(0.90)	(2.96)	(4.25)	(2.70)	(5.67)
HXZ + MOM	0.08	−0.25	−0.08	−0.21	−0.30	−0.08	−0.06	0.08	0.40	0.74	0.66	1.16
+BAB	(0.55)	(−1.98)	(−0.71)	(−2.41)	(−3.27)	(−0.80)	(−0.52)	(0.71)	(2.82)	(4.15)	(2.53)	(5.62)
HXZ + MOM	−0.17	−0.27	−0.10	−0.14	−0.24	0.03	0.04	0.09	0.56	0.67	0.84	1.30
+ REV +LIQ	(−0.99)	(−2.02)	(−0.82)	(−1.38)	(−2.32)	(0.25)	(0.34)	(0.69)	(3.59)	(3.43)	(2.88)	(5.75)

Table V

Four- and Six-Factor Loadings of Portfolios Sorted on Relative Model Value

Portfolios are sorted as in Table II (decile 1 = most overvalued, decile 10 = most undervalued). The table shows the portfolios' loadings from a four- and a six-factor model. The factors in the four-factor model are the Fama and French (1993) factors augmented with a momentum factor. The factors in the six-factor model are the Fama and French (2015) factors augmented with a momentum factor. t-statistics are in parentheses. The sample period is 1983 to 2012.

	Decile										
	1	2	3	4	5	6	7	8	9	10	10−1
Panel A: Four-Factor Loadings											
MKTmRF	1.258	1.151	1.017	0.982	0.925	0.905	0.904	0.890	0.948	0.968	−0.290
	(34.07)	(42.49)	(39.54)	(45.32)	(42.17)	(37.30)	(36.49)	(34.92)	(32.14)	(25.09)	(−5.07)
SMB	0.333	−0.055	−0.138	−0.077	−0.011	0.002	−0.020	0.058	0.165	0.476	0.143
	(6.29)	(−1.40)	(−3.75)	(−2.48)	(−0.36)	(0.06)	(−0.56)	(1.57)	(3.90)	(8.60)	(1.74)
HML	−0.261	−0.157	−0.036	0.154	0.322	0.310	0.418	0.369	0.458	0.364	0.625
	(−4.67)	(−3.83)	(−0.93)	(4.70)	(9.72)	(8.45)	(11.17)	(9.58)	(10.28)	(6.25)	(7.22)
MOM	0.049	0.034	0.027	−0.001	−0.043	−0.092	−0.127	−0.148	−0.170	−0.466	−0.515
	(1.42)	(1.36)	(1.15)	(−0.03)	(−2.09)	(−4.07)	(−5.51)	(−6.26)	(−6.19)	(−12.99)	(−9.67)
Panel B: Six-Factor Loadings											
MKTmRF	1.139	1.119	1.067	1.046	0.993	0.980	0.970	0.943	0.980	0.986	−0.153
	(32.76)	(38.54)	(38.65)	(47.72)	(45.12)	(39.55)	(38.33)	(36.19)	(31.15)	(23.48)	(−2.69)
SMB	0.121	−0.129	−0.103	0.007	0.075	0.079	0.074	0.167	0.254	0.549	0.428
	(2.44)	(−3.10)	(−2.60)	(0.23)	(2.40)	(2.22)	(2.05)	(4.47)	(5.64)	(9.15)	(5.27)
HML	0.014	−0.096	−0.189	−0.014	0.140	0.100	0.247	0.257	0.406	0.349	0.336
	(0.21)	(−1.72)	(−3.56)	(−0.34)	(3.31)	(2.10)	(5.09)	(5.12)	(6.72)	(4.33)	(3.07)
MOM	0.105	0.049	0.004	−0.031	−0.075	−0.127	−0.159	−0.174	−0.185	−0.475	−0.580
	(3.49)	(1.96)	(0.16)	(−1.62)	(−3.93)	(−5.91)	(−7.21)	(−7.67)	(−6.78)	(−13.02)	(−11.76)
RMW	−0.708	−0.233	0.160	0.304	0.316	0.298	0.333	0.353	0.271	0.213	0.921
	(−11.09)	(−4.38)	(3.17)	(7.57)	(7.83)	(6.55)	(7.17)	(7.38)	(4.70)	(2.77)	(8.83)
CMA	−0.252	−0.016	0.260	0.220	0.247	0.319	0.212	0.071	−0.023	−0.077	0.175
	(−2.68)	(−0.20)	(3.48)	(3.72)	(4.15)	(4.76)	(3.10)	(1.00)	(−0.27)	(−0.67)	(1.14)

portfolio loads positively on the value factor and negatively on the momentum factor. A positive loading on the value factor means that our classification of stocks bears similarity to the traditional classification of value versus growth. Although we still find an alpha in models that include the value factor in Table IV, we show in Section III.B that we continue to find an alpha in double-sorted portfolios where the second sorting variable is value classified using book-to-market or a metric using residual income valuation. We also show in Section III.D that we find a statistically significant coefficient on relative value in Fama and MacBeth (1973) regressions in the presence of the book-to-market ratio.

Panel B of Table V shows the loadings of decile portfolios on the six-factor model that includes the five Fama and French (2015) factors and the momentum factor. Recall from Table IV that the alpha of 0.91% from the four-factor model is similar to 0.52% from the six-factor model. Nevertheless, Panel B shows that decile 1 (10) stocks load negatively (positively) on the profitability RMW factor. Decile 1 stocks also load negatively on the investment CMA factor although the difference in loadings of deciles 1 and 10 on this factor is not statistically significant. In other words, overvalued stocks are similar to unprofitable, high investment stocks, while undervalued stocks are similar to profitable stocks—patterns similar to those in the descriptives in Table II.

Note also that decile 1 overvalued stocks are high beta stocks relative to decile 10 undervalued stocks. It thus seems that part of the compensation to the 10 − 1 portfolio returns is due to the loading on the BAB factor. We check loadings on the BAB factor in a seven-factor model that includes the five Fama and French (2015) factors, the momentum factor, and the BAB factor. As expected, we find negative (positive) statistically significant BAB loadings for decile 1 (10). The 10 − 1 portfolio has a loading of 0.219 on the BAB factor. Nevertheless, Table IV shows no appreciable difference in alphas due to the inclusion of the BAB factor (the alpha for FF5+MOM and FF5+MOM+BAB is 0.52% and 0.47%, respectively).

Overall, the results in this subsection show that our strategy loads strongly on well-known anomalies of size, value, momentum, and profitability in an interesting way. Overvalued stocks are closer to small, value, past-return loser, and unprofitable stocks than are undervalued stocks. Many of these characteristics are related to the relevance of the option to default. In Section III.C, we analyze some of these characteristics in more detail.

A.3. Robustness Checks

We perform a battery of additional robustness checks. Relegating the detailed results to Internet Appendix Section I, here we briefly discuss the tests that we perform.

Boguth et al. (2016) find linkages in average returns calculated at different horizons and document severe horizon effects in many style portfolios. We follow their lead in calculating alphas from factor models with additional lags of factors (see also Dimson (1979) and Scholes and Williams (1977) for earlier literature on market beta estimation with lags of the market factor). We find that our portfolio alphas from factor models with lags of factors are very similar to, and in many cases even greater than, the portfolio alphas from standard models. We also compound returns to different frequencies and calculate alphas from these compounded returns with no discernible effect on returns.

In recent work, Boguth et al. (2011) and Cederburg and O'Doherty (2016) implement conditional tests of asset pricing and find that accounting for market timing and volatility timing can have significant effects on alpha estimates. To gauge the seriousness of this issue, we run the conditional asset pricing tests with conditioning variables dividend yield, term spread, default spread, market volatility, and lagged betas. The results are again very similar to those reported in Table III.

We further examine the robustness of the results to different kinds of subsamples, and holding periods up to 18 months. The results from these alternative specifications are very similar to those in Table III.

B. Is It the Value Premium in Disguise?

We check that our results are not just a manifestation of the value effect in stock returns. This concern is warranted as both our relative value measure and traditional value measures compare a fundamental value to actual market value. The results in Table III already show that the alpha of the long-short relative value portfolio remains significant in the presence of the HML factor, which indicates that our measure goes deeper than the simple value effect. Likewise, we find positive and significant coefficients on the relative valuation measure while controlling for the market-to-book ratio in the Fama and MacBeth (1973) regressions reported in Section III.D below. To provide additional evidence, we examine model performance separately for portfolios sorted by market-to-book ratio. Specifically, each month we first sort all stocks into five equal-sized quintiles according to the market-to-book ratio, using the current market value and the book value of the previous quarter. Then, within each market-to-book quintile, we sort all stocks into five equal-sized portfolios by the ratio of model value to market value. These second sorted portfolios are labeled R1 (most overvalued) to R5 (most undervalued).

Panel A of Table VI reports the results of this exercise. There is no clear relation between the relative default option value and market-to-book, although firms in mid-market-to-book quintiles tend to have slightly lower default option values. More importantly, while there is some variation in the four-factor alphas produced by the long-short strategy, the performance is strong and significant in all market-to-book quintiles.

It is also instructive to compare our valuation measure with other valuation measures such as those from the residual income model. We adopt the framework of Frankel and Lee (1998) and calculate the residual income value of firm i in month t as

$$V_{it} = B_{it} + \frac{FROE_{it+1} - Re_i}{1 + Re_i} B_{it} + \frac{FROE_{it+2} - Re_i}{(1 + Re_i)Re_i} B_{it+1}, \qquad (14)$$

Table VI

Four-Factor Alphas of Portfolios Double-Sorted on Relative Model Value and Market-to-Book Ratio or Residual Income Model Value to Price Ratio

Each month, we first sort all stocks into quintiles based on the market-to-book ratio (in Panel A) or residual income model value to price ratio (in Panel B). Market-to-book is calculated as the ratio of the current market value divided by the book value of the previous quarter; MB1 = highest market-to-book; MB5 = lowest market-to-book. Residual income model value is calculated based on Frankel and Lee (1998); VP1 = most overvalued, VP5 = most undervalued. The stocks are then further sorted into quintiles according to the ratio of the equity value implied by our valuation model to actual equity value (R1 = most overvalued, R5 = most undervalued). The holding period for all portfolios is one month. For each MB/VP quintile, we report value-weighted alphas of all relative value portfolios, and the equal-weighted alpha of the hedge portfolio. Alphas are calculated from a four-factor model where the factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding *t*-statistics are in parentheses. The sample period is 1983 to 2012.

	Quintile						ew
	R1	R2	R3	R4	R5	R5−R1	R5−R1
Panel A: Market-to-Book Ratio							
MB1	−0.24	0.16	0.45	0.33	0.85	1.08	0.73
	(−0.98)	(0.86)	(2.41)	(1.59)	(2.83)	(2.79)	(2.77)
MB2	−0.03	0.31	0.16	0.64	0.62	0.65	0.41
	(−0.19)	(2.31)	(1.07)	(5.43)	(3.48)	(2.53)	(2.50)
MB3	−0.12	−0.07	0.08	0.31	0.51	0.63	0.44
	(−0.76)	(−0.53)	(0.63)	(2.50)	(3.22)	(2.81)	(2.85)
MB4	−0.30	−0.04	0.03	0.03	0.26	0.57	0.60
	(−1.92)	(−0.35)	(0.23)	(0.26)	(1.83)	(2.83)	(3.72)
MB5	−0.39	−0.07	0.11	0.12	0.23	0.62	0.80
	(−1.75)	(−0.43)	(0.95)	(1.04)	(1.80)	(2.25)	(3.84)
Panel B: Residual Income Model Value to Price Ratio							
VP1	−0.35	−0.16	−0.35	0.03	0.35	0.70	0.78
	(−1.34)	(−0.66)	(−1.57)	(0.14)	(1.33)	(1.94)	(2.67)
VP2	−0.02	0.28	−0.05	−0.32	0.40	0.42	0.37
	(−0.09)	(1.59)	(−0.31)	(−2.08)	(2.28)	(1.72)	(2.06)
VP3	−0.19	0.13	0.00	0.27	0.42	0.61	0.49
	(−1.22)	(0.95)	(0.01)	(1.77)	(2.59)	(2.88)	(3.00)
VP4	−0.07	−0.08	0.06	0.29	0.44	0.50	0.43
	(−0.41)	(−0.56)	(0.48)	(1.98)	(2.72)	(2.37)	(2.79)
VP5	−0.07	0.17	0.29	0.36	0.56	0.63	0.42
	(−0.37)	(1.14)	(1.84)	(2.26)	(2.71)	(2.31)	(2.42)

where B_t is the book value of equity from the most recent financial statement, Re is the equity cost of capital estimated by the CAPM over the past 36 months, $FROE_{t+1}$ and $FROE_{t+2}$ are the forecasted return on equity for the next two years obtained from IBES, and $B_{t+1} = B_t(1 + (1 - k)FROE_{t+1})$, where k is the dividend payout ratio computed as the most recent annual dividend paid divided by net income before extraordinary items. We then again analyze double-sorted portfolios where the first sorting variable is the residual income model value to price and the second sorting variable is our relative valuation measure. We label the residual income model value to price ratio quintiles by VP, where VP1 is the quintile with the most overvalued stocks and VP5 is the quintile with the most undervalued stocks. Panel B of Table VI reports the results of this exercise. Once again, we see that the four-factor alphas produced by the long-short strategy are strong and significant in all VP quintiles.

Since double-sorts are akin to orthogonalizing our measure against other value measures, the results in this subsection demonstrate that our valuation measure is not subsumed by traditional valuation measures and goes beyond the simple value effect.[19]

C. The Importance of Default Option in Model Valuation

Our valuation model is inspired by the option-like characteristics of common stocks. The option value can be a significant fraction of the total value of equity for stocks depending on the extent of financial distress they face, as well as their size, volatility, and profitability. In this subsection, we verify the importance of this default option for the ability of the model to value stocks and thereby predict returns among these subgroups of stocks.

Perhaps, the most natural characteristic that one can associate with the relevance of the option to default is the extent of financial distress. Indeed, the terms "financial distress" and "high default risk" are often used interchangeably: firms experiencing financial distress are associated with more uncertainty about their ability to generate sufficient future cash flows, thus making the option to default particularly relevant for them. Put differently, for highly distressed firms, the option to default is likely to be in-the-money and capture a significant fraction of total equity value. We expect our model's ability to detect misvaluation to be higher among financially distressed stocks. We employ the model of CHS (2008) to measure financial distress.

The second characteristic that we consider is firm size. Since firm size is one input in the CHS (2008) distress measure, one can view firm size as a reduced-form proxy for the likelihood of default. Also, in general, young and small firms face more competitive challenges and higher capital constraints and therefore are more likely to default or abandon their business. We measure firm size by equity market value and expect our model to perform better for small-cap stocks.

The third firm characteristic we consider is stock return volatility. High uncertainty about the future of firms facing the possibility of default is likely to be reflected in high stock return volatility. In particular, any news about future cash flows that affects the likelihood that the firm will default will also affect the current price. In turn, as implied by option pricing theory, the value of an option increases with the volatility of the underlying asset. We

[19] We also note that our model neither is designed to explain nor does manage to explain the value premium.

use idiosyncratic volatility as our main volatility measure (we obtain similar results using total volatility instead). We follow Ang et al. (2006) and measure idiosyncratic volatility each month by the standard deviation of the residuals of a regression of daily stock returns on the daily Fama and French (1993) three factors augmented with the momentum factor. For each month, idiosyncratic volatility is estimated during the previous month.[20] We expect better model performance for highly volatile stocks.

The fourth and last characteristic is profitability. We use the ratio of net income to total assets (ROA) as a proxy for profitability. We expect less profitable companies to be closer to the default boundary and hence face a higher default probability. Like size, this ratio is one of the inputs in the CHS (2008) model.

We proceed as follows. Each month we first sort all stocks into five quintiles according to each characteristic, using current market data and quarterly accounting data from the previous quarter. We use equal-sized quintiles for distress, volatility, and profitability and quintiles based on NYSE breakpoints for size. Then, within each characteristic quintile, we sort all stocks into five equal-sized portfolios according to the ratio of the model value to the market value. These second sorted portfolios are labeled R1 (most overvalued) to R5 (most undervalued). Our double-sorted portfolios are well populated, as the average number of stocks per portfolio is 123.

We report the mean fraction of the default option value in the total equity value implied by our valuation model for value-weighted portfolios in the last column of each characteristic panel in Table VII. To compute this fraction, we run the model while shutting down the default option by disallowing default and exit and forcing equity holders to operate the firm indefinitely. The value of the option to default is then given as the difference in equity values with and without this option (see equation (13)). These mean fractions confirm our choice of characteristics, as they increase with distress and volatility, and decrease with firm size. For example, the value of the option to default is, on average, 35.9% of the total value of equity for the most distressed stocks, but only 19.2% of the total value of equity for the least distressed stocks. Note that our model captures both the default and the abandonment options and therefore implies positive option values for even zero-leveraged firms as long as fixed costs are nonzero. The mean fraction is increasing from 15.4% to 30.1% when moving from low- to high-volatility stocks. Size and profitability have a weaker effect on the default option. The mean fractions are 28.6% and 17.2% for the large- and small-size quintiles, respectively, and decrease from 27.6% for the least profitable firms to 20.5% for the most profitable firms. High profitability firms might optimally choose more debt financing, which would make the default option more important. This effect may, in turn, attenuate the relation between profitability and the relative value of the option to default.

Table VII also shows the four-factor alphas for these double-sorted portfolios. For each characteristic sort we, find that (i) the four-factor alphas for R5 (most

[20] The results remain similar when we take the average idiosyncratic volatility over the prior 3 months or over the prior 12 months.

Table VII

Four-Factor Alphas of Portfolios Double-Sorted on Relative Model Value and Characteristics Related to Default Option

Each month, we first sort all stocks into quintiles based on a stock characteristic related to the default option. The stocks are then further sorted into quintiles according to the ratio of the equity value implied by our valuation model to actual equity value (R1 = most overvalued, R5 = most undervalued). The variable for the first sort is distress in Panel A, size in Panel B, stock return idiosyncratic volatility in Panel C, and profitability in Panel D. Distress is calculated based on CHS (2008) using current market data and quarterly accounting data for the most recent available quarter. Size is market capitalization. Idiosyncratic volatility is calculated as the standard deviation of the residuals of a regression of daily stock returns on the daily four factors over the last month. Profitability is given as the ratio of net income to total assets, calculated using quarterly data. The holding period for all portfolios is one month. For each characteristic quintile, we report value-weighted alphas of all relative value portfolios and the equal-weighted alpha of the hedge portfolio. Alphas are calculated from a four-factor model where the factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding *t*-statistics are in parentheses. The last column in each panel gives the fraction of value coming from the default option (in percent). To compute this fraction, we run the model while shutting down the default option (i.e., imposing the restriction that the firm is run by equity holders indefinitely). The value of the option to default is then given as the difference in equity values with and without this option. The sample period is 1983 to 2012.

	Quintile						ew	
	R1	R2	R3	R4	R5	R5−R1	R5−R1	DefOpt
				Panel A: Distress				
D1	0.18	0.14	0.14	0.04	0.43	0.26	0.39	19.2
	(1.41)	(1.21)	(1.18)	(0.27)	(2.81)	(1.38)	(2.92)	
D2	−0.26	0.02	0.20	0.30	0.37	0.63	0.52	16.6
	(−2.07)	(0.22)	(1.80)	(2.40)	(2.95)	(3.45)	(3.68)	
D3	−0.26	−0.01	0.00	0.50	0.53	0.78	0.81	19.2
	(−1.60)	(−0.08)	(0.01)	(4.06)	(3.40)	(3.27)	(5.36)	
D4	−0.45	−0.53	−0.30	0.11	0.36	0.81	1.04	22.3
	(−1.83)	(−2.75)	(−1.81)	(0.57)	(1.67)	(2.49)	(5.42)	
D5	−0.85	−1.19	−0.21	0.65	0.34	1.19	1.81	35.9
	(−2.39)	(−3.69)	(−0.62)	(1.98)	(0.69)	(2.10)	(6.12)	
				Panel B: Size				
S1	−0.44	0.14	0.15	0.35	0.31	0.75	1.19	28.7
	(−2.95)	(1.21)	(1.54)	(3.21)	(2.09)	(4.31)	(7.08)	
S2	−0.11	0.03	0.17	0.26	0.22	0.34	0.33	22.6
	(−0.90)	(0.33)	(1.78)	(2.81)	(1.95)	(1.84)	(1.82)	
S3	−0.22	−0.19	−0.06	0.21	0.47	0.69	0.66	19.8
	(−1.72)	(−1.98)	(−0.66)	(2.06)	(4.07)	(3.80)	(3.64)	
S4	−0.10	0.10	−0.06	0.17	0.42	0.52	0.55	17.4
	(−0.73)	(0.91)	(−0.60)	(1.69)	(3.50)	(2.85)	(2.96)	
S5	−0.10	−0.01	0.05	−0.03	0.33	0.43	0.43	17.2
	(−0.80)	(−0.13)	(0.55)	(−0.34)	(2.90)	(2.30)	(2.42)	

(Continued)

Table VII—*Continued*

	Quintile						ew	
	R1	R2	R3	R4	R5	R5−R1	R5−R1	DefOpt
	Panel C: Idiosyncratic Volatility							
IV1	−0.05	0.03	0.13	0.20	0.34	0.39	0.29	15.4
	(−0.41)	(0.28)	(1.36)	(2.01)	(2.80)	(2.31)	(2.47)	
IV2	−0.11	0.11	−0.03	0.09	0.48	0.59	0.59	19.4
	(−0.80)	(0.86)	(−0.25)	(0.61)	(3.21)	(2.78)	(4.25)	
IV3	−0.01	−0.06	0.06	0.24	0.85	0.86	0.73	22.0
	(−0.06)	(−0.31)	(0.36)	(1.35)	(4.20)	(2.96)	(4.29)	
IV4	−0.52	−0.35	0.13	0.45	0.23	0.75	1.23	25.9
	(−1.96)	(−1.50)	(0.56)	(2.01)	(0.94)	(2.12)	(5.94)	
IV5	−1.08	−0.11	−0.65	−0.31	0.30	1.38	1.82	30.1
	(−2.91)	(−0.34)	(−2.59)	(−1.03)	(0.80)	(2.80)	(6.66)	
	Panel D: Profitability							
PR1	−0.87	−0.31	0.10	−0.10	0.21	1.09	1.81	27.6
	(−2.96)	(−1.09)	(0.38)	(−0.42)	(0.75)	(2.72)	(6.38)	
PR2	−0.17	−0.53	−0.11	0.13	0.36	0.54	0.66	21.3
	(−0.93)	(−3.31)	(−0.82)	(0.85)	(1.70)	(1.79)	(2.84)	
PR3	−0.17	−0.17	0.07	0.35	0.41	0.58	0.79	15.0
	(−1.25)	(−1.44)	(0.66)	(2.82)	(2.73)	(2.90)	(4.99)	
PR4	−0.19	−0.02	−0.06	0.14	0.41	0.60	1.05	16.5
	(−1.45)	(−0.15)	(−0.51)	(1.05)	(2.82)	(3.06)	(6.73)	
PR5	0.05	0.33	0.22	0.44	0.74	0.69	0.86	20.5
	(0.36)	(2.46)	(1.70)	(2.92)	(4.24)	(3.12)	(4.78)	

undervalued stocks) are always higher than those for R1 (most overvalued stocks), and (ii) the hedge portfolio alphas are increasing with the value of the default option. For example, Panel A shows that the hedge portfolio alpha is only 0.26% (t-statistic = 1.38) for the least distressed quintile of stocks and increases monotonically to 1.19% (t-statistic = 2.10) for the most distressed quintile of stocks. The effect of size in Panel B on the returns generated by the model's relative value portfolios is somewhat weaker than those of distress: the four-factor alpha of the hedge portfolio is 0.75% for the small size quintile and 0.43% for the large size quintile. These return patterns are consistent with the weaker relation between size and the fraction of the default option shown in the last column of Panel B. Panel C shows that the four-factor alpha of the hedge portfolio is 1.38% for high-volatility stocks (for which the default option is more valuable), whereas it is only 0.39% for low-volatility stocks (where the default option is less valuable). Panel D shows that the hedge portfolio alpha drops from 1.09% for the least profitable stocks to 0.69% for the most profitable stocks.

Reflecting the better performance of the model for small stocks, equal-weighted hedge portfolio returns in Table VII show stronger support for our conjecture. The equal-weighted portfolio return difference between stocks with high and low default option value is greater than that for value-weighted portfolios. For instance, Panel B shows that the four-factor alpha of the equal-weighted hedge portfolio is 1.19% for the small size quintile and only 0.43% for the large size quintile. The four-factor alpha is 1.81% per month (t-statistic = 6.12) for the most distressed D5 stocks in Panel A, 1.82% (t-statistic = 6.66) for the most volatile IV5 stocks in Panel C, and 1.81% (t-statistic = 6.38) for the least profitable PR1 stocks in Panel D, consistent with the fraction of the default option being highest among these categories of stocks.

The returns in Table VII show that the strength of the model in valuing stocks is largely driven by the option to default. To provide a more direct test of the importance of the default option in total equity value, we recompute the returns to our double-sorted portfolios by shutting down the default option. Table VIII shows the four-factor alphas to the long-short R5 − R1 portfolios for each characteristic quintile. The performance of the model in predicting returns deteriorates sharply without the option to default. For the most distressed stocks and for stocks with the greatest idiosyncratic volatility, the four-factor alpha of the model without the default option is roughly a quarter of the magnitude of the alpha of the model with the default option. In particular, there is a reduction in four-factor alpha from 1.19% to 0.28% for the most distressed stocks, and from 1.38% to 0.33% for the most volatile stocks. For the least profitable stocks, the alpha of the long-short strategy drops from 1.09% with the default option to 0.48% without the default option. In contrast, the reduction in alpha is relatively modest for small stocks. The deterioration in model performance is relatively more modest for equal-weighted portfolio returns in Panel B.

D. Fama-MacBeth Regressions on Relative Model Value

The portfolio sorts provide a simple view of the relation between returns and our variables of interest. Another approach commonly used in the literature is Fama and MacBeth (1973) regressions. Beyond serving as an additional diagnostic check, these regressions offer the advantage of controlling for other well-known determinants of the cross-sectional patterns in returns and thus check for the marginal influence of relative model valuation on our results. Accordingly, we run these cross-sectional regressions and report the results in Table IX. The dependent variable is the excess stock return, while the independent variables are log market capitalization (*SZ*), log market-to-book (*MB*), past six-month return (*Ret6*), relative model value calculated as the log of the ratio of the equity value implied by our valuation model to actual equity value (*MODEL*, where higher numbers indicate undervaluation based on our model), the CHS (2008) distress risk measure (*DIST*), volatility (*IVOL*), profitability (*PROF*), and interaction terms between relative model value and the characteristics of interest. We winsorize all independent variables at the 1% and 99% levels to reduce the impact of outliers. All reported coefficients are multiplied by 100, and we report Newey and West (1987) corrected (with six lags) t-statistics in parentheses.

Table VIII

Four-Factor Alphas of Portfolios Double-Sorted on Relative Model Value (without the Default Option) and Characteristics Related to Default Option

Portfolios are sorted as in Table VII. We run the model two times, first with the default option and then without the default option. The default option is shut down by imposing the restriction that the firm is always run by equity holders. For each characteristic, the left column shows the four-factor alpha of the long-short relative value portfolio R5 − R1 as in Table VII. The right column shows the equivalent alphas based on model values without the option to default. The factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding *t*-statistics are in parentheses. The sample period is 1983 to 2012.

	Distress		Size		Idio Volatility		Profitability	
	with Option	without Option	with Option	without Option	with Option	without Option	with Option	without Option
	Panel A: Value-Weighted Returns							
Q1	0.26	0.19	0.75	0.54	0.39	0.43	1.09	0.48
	(1.38)	(0.87)	(4.31)	(2.98)	(2.31)	(2.37)	(2.72)	(0.78)
Q2	0.63	0.55	0.34	0.42	0.59	0.24	0.54	0.73
	(3.45)	(2.69)	(1.84)	(2.20)	(2.78)	(1.06)	(1.79)	(2.28)
Q3	0.78	0.75	0.69	0.91	0.86	0.67	0.58	0.67
	(3.27)	(3.09)	(3.80)	(4.93)	(2.96)	(2.12)	(2.90)	(3.38)
Q4	0.81	0.22	0.52	0.37	0.75	0.45	0.60	0.83
	(2.49)	(0.60)	(2.85)	(1.91)	(2.12)	(1.06)	(3.06)	(3.91)
Q5	1.19	0.28	0.43	0.34	1.38	0.33	0.69	0.16
	(2.10)	(0.41)	(2.30)	(1.76)	(2.80)	(0.54)	(3.12)	(0.62)
	Panel B: Equal-Weighted Returns							
Q1	0.39	0.22	1.19	0.87	0.29	0.48	1.81	1.12
	(2.92)	(1.33)	(7.08)	(4.71)	(2.47)	(4.19)	(6.38)	(2.65)
Q2	0.52	0.51	0.33	0.42	0.59	0.40	0.66	0.95
	(3.68)	(3.25)	(1.82)	(2.25)	(4.25)	(2.96)	(2.84)	(4.29)
Q3	0.81	1.01	0.66	0.90	0.73	0.70	0.79	0.94
	(5.36)	(6.40)	(3.64)	(4.93)	(4.29)	(3.66)	(4.99)	(5.74)
Q4	1.04	0.99	0.55	0.39	1.23	1.07	1.05	0.94
	(5.42)	(4.42)	(2.96)	(2.03)	(5.94)	(3.71)	(6.73)	(6.22)
Q5	1.81	1.30	0.43	0.26	1.82	1.26	0.86	0.71
	(6.12)	(3.18)	(2.42)	(1.28)	(6.66)	(3.19)	(4.78)	(3.53)

The first regression shows the usual patterns—returns are related to size, market-to-book, and past return. The second regression is the version of the univariate sorts in Table III, showing that our valuation measure is able to predict returns. Regression (3) shows that distress and idiosyncratic volatility are negatively related to returns, while profitability is positively related, although the statistical significance of the volatility measure is not high for our sample period. Specifications (4) and (5) show that relative model value is positively associated with future returns even after the inclusion of standard

Table IX

Fama-MacBeth Regressions on Relative Model Value

Each month we run cross-sectional Fama and MacBeth (1973) regressions of excess stock returns. The independent variables are log market capitalization (*SZ*), log market-to-book (*MB*), past six-month return (*Ret6*), distress risk measure (*DIST*), idiosyncratic volatility (*IVOL*), profitability (*PROF*), and relative model value (*MODEL*). *MB* is calculated as the ratio of current market value divided by book value in the previous quarter. We skip one month in calculating *Ret6*. *DIST* is calculated based on CHS (2008) using current market data and quarterly accounting data for the previous quarter. *IVOL* is calculated as the standard deviation of the residuals of the regression of daily stock returns on the daily four factors over the past month. *PROF* is given as the ratio of net income to total assets, calculated using quarterly data. *MODEL* is the log of the ratio of the equity value implied by our valuation model to actual equity value; higher numbers indicate undervaluation based on our model. All coefficients are multiplied by 100 and Newey and West (1987) corrected *t*-statistics (with six lags) are in parentheses. The sample period is 1983 to 2012.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
CNST	1.575	0.858	1.298	1.631	1.378	1.660	0.576	2.459	1.962	1.421
	(1.95)	(2.66)	(1.68)	(2.01)	(1.82)	(2.04)	(0.50)	(5.15)	(2.64)	(1.92)
SZ	−0.068		−0.156	−0.072	−0.147	−0.076	−0.133	−0.119	−0.097	−0.154
	(−1.35)		(−5.19)	(−1.41)	(−4.85)	(−1.47)	(−3.89)	(−3.53)	(−2.14)	(−5.22)
MB	−0.166		−0.212	−0.097	−0.151	−0.097	−0.163	−0.091	−0.127	−0.152
	(−2.64)		(−3.79)	(−1.75)	(−3.22)	(−1.75)	(−3.11)	(−1.84)	(−2.34)	(−3.27)
Ret6	0.278		0.137	0.306	0.180	0.292	0.146	0.337	0.250	0.166
	(1.46)		(0.76)	(1.64)	(1.02)	(1.58)	(0.82)	(1.90)	(1.35)	(0.95)
MODEL		0.217		0.194	0.153	0.710	0.599	0.046	0.144	0.684
		(4.46)		(4.41)	(4.35)	(3.95)	(4.86)	(0.76)	(3.60)	(2.41)
SZ × *MODEL*						−0.048				−0.036
						(−2.85)				(−2.15)
DIST			−0.198		−0.171		−0.238			−0.174
			(−2.83)		(−2.55)		(−2.46)			(−2.62)
DIST × *MODEL*							0.069			0.029
							(4.13)			(1.50)
IVOL			−6.282		−5.858			−12.598		−5.940
			(−1.15)		(−1.07)			(−1.78)		(−1.10)
IVOL × *MODEL*								3.498		0.461
								(2.38)		(0.25)
PROF			2.022		1.484				3.014	1.031
			(2.24)		(1.74)				(2.28)	(1.05)
PROF × *MODEL*									−0.359	−0.003
									(−1.44)	(−0.01)

stock characteristics. These regressions provide strong multivariate evidence of the efficacy of our valuation measure and hence corroborate the portfolio sort results.

Next, we interact our valuation measure with the stock characteristics to gauge whether our model works better for stocks with a high default option value. Specifications (6) through (10) are regression counterparts of Table VII. Regressions (6) to (8) show that the interaction terms between relative model value and size, distress, and idiosyncratic volatility are statistically significant. This implies that our relative model value does particularly well for small stocks, distressed stocks, and more volatile stocks. It is important to note that the effect of the interaction term on size in specification (6) is highly statistically significant in contrast to the somewhat weaker results from the portfolio sorts. The coefficient on the interaction term with profitability has the expected negative sign though it is statistically insignificant. As discussed above, the effect of profitability might be subdued by the endogeneity of financial leverage—more profitable firms might optimally choose higher leverage, which would increase the likelihood of default. Finally, including all variables and interaction terms in regression (10), we find that the coefficients on relative model value and the interaction term with size are significant although the coefficients on other interaction terms lose their significance.

To summarize, the cross-sectional regressions of Table IX together with the portfolio sort results provided in Tables VII and VIII demonstrate the importance of the default option in the model valuation. Both portfolio sort results and regression-based evidence suggest that the power of the model is stronger for stocks with higher default option values and characteristics closely associated with financial distress and default.

IV. Additional Tests for Mispricing

As mentioned in Section II, extant literature explores equity misvaluation by comparing market values to the values derived from equity valuation models such as the dividend discount model or the residual income model. We follow in the footsteps of these papers using a relatively more sophisticated model (albeit a relatively standard model from the real options literature). Along the way, we have to make a number of assumptions to implement the model empirically. This might raise concerns about our model. The fact that we see large spreads in future returns (and factor model alphas) to over- versus undervalued stocks suggests to us that our model does a good job identifying mispriced stocks, especially stocks that are hard to value. If the market prices were mostly correct and our model had a lot of noise, we would not see these spreads. In this section, we undertake additional tests to demonstrate that it is indeed market mispricing that is driving our results and that a richer rational model would not likely overturn them.

A. Earnings Announcements

We follow the literature in examining what might be triggers for market learning that would eventually correct the mispricing. In particular, we examine stock returns around earnings announcements after portfolio formation (see, e.g., Lakonishok, Shleifer, and Vishny (1994), La Porta et al. (1997), and Cooper, Gulen, and Schill (2008)). If mispricing drives our results, then return spreads around EADs should be higher than those around non-EADs.

We obtain EADs from quarterly Compustat (data item RDQE). In the first test, we partition the sample into firm months with and without EADs. Panel A of Table X shows four-factor alphas for deciles sorted on relative model value for these subsamples. While long-short 10 − 1 portfolio alphas in both samples are statistically significant, the magnitude of 1.32% in months when firms had EADs is almost twice that of 0.67% in months with no EADs. To gain even more precision, in Panel B, we look at daily returns in a three-day window around EADs. Once again, the 10 − 1 portfolio four-factor alphas are 0.24% per day around EADs but only 0.05% around non-EADs. Equal-weighted portfolios show even larger differences in alphas in both panels. These results suggest that subsequent earnings announcements impart valuable information to the market and are consistent with expectational errors in stock pricing.

B. Information Transparency

Market mispricing is likely to be stronger among stocks with less information transparency. To examine the potential effect of mispricing on the long-short portfolio returns, we use three proxies for the degree of information transparency. Our first proxy is analyst coverage. Existing literature argues that analyst coverage is associated with information production (see, e.g., Chang, Dasgupta, and Hilary (2006), Kelly and Ljungqvist (2012), and Derrien and Kecskés (2013)). We obtain analyst coverage from IBES.

Our second proxy for information transparency is institutional ownership. We follow the literature that shows that greater institutional ownership enhances information transparency. See, for example, Bushee and Noe (2000), Ajinkya, Bhojraj, and Sengupta (2005), and Boon and White (2015) for evidence of an association between institutional ownership and management disclosure, information asymmetry, as well as the quality and frequency of management forecasts. Institutional ownership is taken from the Thomson 13F institutional holding database.

Our third and last proxy for information transparency is the availability of listed options for a given stock. Jennings and Starks (1986) and Senchack and Starks (1993) show that the prices of stocks with listed options adjust much faster to new information than the prices of stocks without options. In addition, there is abundant evidence in the literature that the information contained in option prices has predictive ability for subsequent stock returns, suggesting that the options market produces incremental information and

Table X

Four-Factor Alphas of Portfolios Sorted on Relative Model on Earnings Announcement Days

Portfolios are sorted as in Table II (decile 1 = most overvalued, decile 10 = most undervalued). Alphas are calculated from a four-factor model where the factors are market, size, book-to-market, and momentum. The first row shows the same alphas as in Table III. Panel A divides the sample into months in which the firm had an earnings announcement or not. Alphas in this panel are in percent per month. Panel B divides the sample into three-day windows around earnings announcement or not. Alphas in this panel are in percent per day. The corresponding *t*-statistics are in parentheses. The sample period is 1983 to 2012.

	Decile											ew
	1	2	3	4	5	6	7	8	9	10	10−1	10−1
Full sample	−0.24	−0.29	0.08	0.02	−0.07	0.13	0.18	0.26	0.48	0.67	0.91	1.27
	(−1.54)	(−2.43)	(0.67)	(0.16)	(−0.75)	(1.29)	(1.68)	(2.34)	(3.74)	(3.95)	(3.68)	(6.65)
Panel A: Firm-Months with and without Earnings Announcement												
With	−0.38	−0.15	−0.11	0.17	0.28	0.20	0.28	0.65	0.63	0.94	1.32	1.92
	(−1.08)	(−0.51)	(−0.31)	(0.65)	(0.83)	(0.76)	(1.07)	(2.05)	(1.94)	(3.34)	(2.85)	(5.04)
Without	−0.44	−0.48	−0.34	−0.32	−0.51	−0.19	−0.09	−0.02	0.42	0.22	0.67	0.97
	(−2.36)	(−3.38)	(−3.13)	(−2.79)	(−4.05)	(−1.71)	(−0.68)	(−0.18)	(2.85)	(1.17)	(2.53)	(4.45)
Panel B: Firm-Days with and without Earnings Announcement												
With	−0.12	0.02	0.03	0.01	0.17	0.07	0.05	0.10	0.09	0.12	0.24	0.40
	(−2.37)	(0.36)	(0.70)	(0.19)	(3.43)	(1.81)	(1.03)	(1.91)	(1.74)	(2.03)	(3.07)	(5.02)
Without	0.00	−0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.11
	(−0.58)	(−2.02)	(−0.35)	(−0.28)	(−0.56)	(0.97)	(1.27)	(2.40)	(3.82)	(4.47)	(3.61)	(11.87)

hence contributes to price discovery.[21] We identify the availability of traded options from 1996 onward using Optionmetrics.

Since we argue that the potential for mispricing is stronger among stocks with low information transparency, we not only expect our model-based investment strategy returns to be higher for stocks with more important default options (as evidenced in Section III.C and Table VII), but also expect the improvement in the performance to be particularly pronounced among stocks with low information transparency. To test this hypothesis, we double-sort all stocks into quintiles based on the characteristics related to default options used in Table VII (CHS (2008) distress score, size, idiosyncratic volatility, and profitability) and into two groups based on the three proxies of information transparency discussed above. For each of these double sorts, we further triple-sort on relative model value and report the value-weighted four-factor alphas of these R5 − R1 relative value portfolios in Table XI. To reduce clutter, we only report the results for quintile portfolios with the most/least important default options.

The results in Table XI provide strong support for our conjecture. The effect of default options on improvement in alphas is greater for stocks with less information transparency, across all proxies for default options and measures of information transparency. For example, as one moves from the least to the most distressed stocks, the strategy's four-factor alpha increases from 0.48% to 2.21% (an improvement of 1.73%) for stocks with low analyst coverage, but from only 0.25% to 0.84% (an improvement of 0.59%) for stocks with high analyst coverage. The corresponding improvement is 2.25% (0.73%) for low (high) institutional ownership and 0.51% (0.29%) for stocks without (with) listed options. The differences in four-factor alphas for the other three proxies for default options (size, volatility, and profitability) exhibit a similar pattern.

C. Time-Series Variation in Model Performance

We next analyze model performance in different states of the economy. Our hypothesis is that valuation difficulties are more pronounced during bad, more turbulent times and times of high investor sentiment. We divide the sample into two parts based on the NBER recession dummy or the Baker and Wurgler (2006) sentiment index.

Table XII shows the four-factor alphas of the 10 − 1 portfolio in these two subsamples. Panel A shows that our valuation model produces a hedge portfolio alpha of 1.61% in recessions and 0.89% in expansions. Note that most overvalued stocks in decile 1 have particularly poor performance in recessions (four-factor alpha of −0.90%). Most undervalued stocks in decile 10, however, perform similarly well in terms of four-factor alphas in both recessions and expansions.

[21] See, for example, Pan and Poteshman (2006), Bali and Hovakimian (2009), Cremers and Weinbaum (2010), and Xing, Zhang, and Zhao (2010).

Table XI

Four-Factor Alphas of Portfolios Triple-Sorted on Relative Model Value, Characteristics Related to Default Option, and Characteristics Related to Information Transparency

Each month, we first sort all stocks into quintiles based on a stock characteristic related to the default option. These characteristics are the same as those in Table VII, namely, distress, size, stock return idiosyncratic volatility, and profitability. The stocks are then further divided into two groups related to information transparency. The characteristics for the second sort are analyst coverage, institutional ownership (IO), and whether the stocks have traded options. Finally, the stocks are sorted into quintiles according to the ratio of the equity value implied by our valuation model to actual equity value (R1 = most overvalued, R5 = most undervalued). The holding period for all portfolios is one month. For each of the first two sorts, we report value-weighted four-factor alphas of R5 − R1 relative value portfolios. The factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding *t*-statistics are in parentheses. The sample period is 1983 to 2012.

		Distress		Size		Idio Volatility		Profitability	
		D1	D5	S1	S5	IV1	IV5	PR1	PR5
Analyst	Low	0.48	2.21	1.35	0.66	0.20	2.17	1.87	0.84
		(1.91)	(4.44)	(6.61)	(3.10)	(0.92)	(4.80)	(4.07)	(3.00)
	High	0.25	0.84	0.49	0.44	0.40	1.20	0.88	0.33
		(1.24)	(1.36)	(2.57)	(2.14)	(2.33)	(2.03)	(2.15)	(1.53)
IO	Low	0.44	2.69	1.51	0.62	0.18	2.59	2.39	0.73
		(1.57)	(3.98)	(6.14)	(2.76)	(0.66)	(4.65)	(4.18)	(2.44)
	High	0.29	1.02	0.37	0.26	0.48	1.37	0.74	0.40
		(1.35)	(1.60)	(1.93)	(1.18)	(2.56)	(2.40)	(1.65)	(1.74)
Options	No	0.65	1.16	0.51	0.79	0.19	1.89	2.02	0.74
		(1.80)	(1.92)	(2.09)	(1.12)	(0.56)	(3.07)	(3.53)	(2.13)
	Yes	0.42	0.71	0.63	0.51	0.61	−0.49	0.59	0.32
		(1.38)	(0.65)	(1.51)	(1.74)	(2.49)	(−0.48)	(0.86)	(0.90)

Panel B shows even bigger differences for the sample split by the sentiment index. The 10 − 1 portfolio alpha is 1.36% in high sentiment months but only 0.52% in low sentiment months. The fact that sentiment plays an important role in mispricing is also consistent with the evidence in Stambaugh, Yu, and Yuan (2012). Overall, these time-series results are suggestive of market mispricing.

D. Characteristics of Undervalued and Overvalued Stocks

The tests in this section so far focus on the relation between hedge portfolio returns and proxies for potential mispricing. As an additional check of the model's ability to identify mispricing, and, in particular, to identify misvaluation of default options, we take a closer look at characteristics of the stocks classified as over- or undervalued by our model. If our model does a reasonable job in detecting mispricing, then such stocks are likely to exhibit characteristics commonly associated with over-/undervaluation. Furthermore, since the model

Table XII

Four-Factor Alphas of Portfolios Sorted on Relative Model Value in Different Time Periods

Portfolios are sorted as in Table II (decile 1 = most overvalued, decile 10 = most undervalued). Alphas are calculated from a four-factor model where the factors are market, size, book-to-market, and momentum. The first row shows the same alphas as in Table III. All alphas are in percent per month and the corresponding *t*-statistics are in parentheses. The full sample period is 1983 to 2012. The sample period is separated into recession and expansion periods based on the NBER recession dummy in Panel A. The sample period is separated into high and low sentiment months based on Baker and Wurgler's (2006) sentiment index in Panel B.

	Decile										
	1	2	3	4	5	6	7	8	9	10	10−1
Full sample	−0.24	−0.29	0.08	0.02	−0.07	0.13	0.18	0.26	0.48	0.67	0.91
	(−1.54)	(−2.43)	(0.67)	(0.16)	(−0.75)	(1.29)	(1.68)	(2.34)	(3.74)	(3.95)	(3.68)
Panel A: Expansions and Recessions											
Recession	−0.90	−0.27	0.07	0.20	0.14	0.43	0.46	0.59	1.15	0.71	1.61
	(−1.70)	(−0.69)	(0.22)	(0.69)	(0.37)	(1.11)	(0.99)	(1.31)	(2.43)	(1.04)	(1.74)
Expansion	−0.19	−0.24	0.06	0.02	−0.11	0.06	0.13	0.21	0.41	0.70	0.89
	(−1.12)	(−1.88)	(0.46)	(0.17)	(−1.07)	(0.58)	(1.27)	(1.93)	(3.06)	(3.98)	(3.42)
Panel B: High and Low Sentiment											
High	−0.63	−0.39	0.08	0.00	−0.05	0.20	0.22	0.29	0.57	0.74	1.36
	(−2.46)	(−1.95)	(0.39)	(0.01)	(−0.29)	(1.02)	(1.22)	(1.53)	(2.83)	(2.52)	(3.29)
Low	0.10	−0.12	0.03	0.06	−0.13	0.11	0.16	0.25	0.35	0.63	0.52
	(0.47)	(−0.73)	(0.21)	(0.54)	(−1.12)	(0.93)	(1.24)	(1.74)	(1.95)	(2.87)	(1.57)

derives its power from incorporating default options explicitly, this association is expected to be stronger for firms with more important default options.

We focus on the following characteristics associated with over-/undervaluation: equity issuance, institutional ownership, insider trading, and the likelihood of becoming an acquisition target. A large body of evidence (see, e.g., Spiess and Affleck-Graves (1995), Loughran and Ritter (1997), and Dong, Hirshleifer, and Teoh (2012)), including survey evidence (Graham and Harvey (2001)), shows that overvalued firms tend to issue more equity. Institutional investors are commonly viewed as more sophisticated and therefore are less likely to hold overvalued stocks. For example, CHS (2008) report that institutional investors tend to sell financially distressed stocks (such stocks exhibit low subsequent returns and are potentially overvalued). It is likely that insiders have superior information about the fundamental value of their firm and it is easier for them to identify potential mispricing and time their trades accordingly. Empirical evidence on insider trading is consistent with this argument (see, e.g., Rozeff and Zaman (1998), Jenter (2005), and Cohen, Malloy, and Pomorski (2012)). Finally, undervalued firms become potentially attractive acquisition targets (see, e.g., Rhodes-Kropf, Robinson, and Viswanathan (2005), Dong et al. (2006), and Malmendier, Opp, and Saidi (2016)).

We obtain information on insider trades from Thomson's Insiders. Net insider buys are calculated as the difference between the number of shares bought and sold by insiders during the portfolio formation month, scaled by total number of shares outstanding. We calculate the likelihood of becoming an acquisition target as the percentage of firms that were acquired during the following year. The data on mergers come from the SDC Platinum database.

Table XIII summarizes the results from this exercise. We double-sort all stocks into quintiles based on a stock characteristic related to default options (distress, size, volatility, and profitability) and relative model value as in Table VI. We then look at the mispricing-related characteristics among firms with high and low default option values separately for the most undervalued and the most overvalued stocks. As before, we argue that the power of the model hinges on incorporating default option values. Therefore, we expect the difference in characteristics chosen in this subsection to be stronger for firms with high default option values.

The results in Table XIII strongly support our hypothesis. Overvalued stocks (quintile R1) are different from undervalued stocks (quintile R5) with respect to their characteristics in the expected direction. While these results are generally consistent with the descriptive statistics in Table II, the novelty of the analysis in this subsection is that the difference in characteristics is larger for those firms for which the default option is more important. For example, the difference in equity issuance between the most overvalued and the most undervalued firms is 4.3% for the most distressed firms, while the same difference is only 1.6% for the least distressed firms. For institutional ownership, insider buys, and takeovers, the corresponding differences are 3.6%, 4.2%, and 2.2%, respectively, for the most distressed firms versus −9.8%, 2.3%, and 0.1%,

Table XIII

Mispricing-Related Characteristics of Portfolios Double-Sorted on Relative Model Value and Characteristics Related to Default Option

Portfolios are double-sorted as in Table VII. The variables for the first sort are distress, size, stock return idiosyncratic volatility, and profitability. The stocks are then further sorted into quintiles according to the ratio of the equity value implied by our valuation model to actual equity value (R1 = most overvalued, R5 = most undervalued). The table presents descriptive statistics for each portfolio, where, for all variables, observations outside the top and bottom percentiles are excluded. For each variable, we first calculate the cross-sectional mean across stocks for each portfolio. We then report the time-series averages of these means. Equity issuance (reported in percent) is measured as the difference between the sale and purchase of common and preferred stocks during the year, scaled by equity market value at the beginning of the year. Institutional ownership (IO, reported in percent) is the sum of all shares held by institutions divided by total shares outstanding. Net insider buys are calculated as the difference between the numbers of shares bought and sold by insiders during the portfolio formation month, scaled by the total number of shares outstanding (reported in basis points). Merger targets is the percentage of firms that were acquired during the following year. The sample period is 1983 to 2012.

		Equity Issuance		IO		Net Insider Buys		Merger Targets	
		R1	R5	R1	R5	R1	R5	R1	R5
Distress	D1	0.7	−0.9	54.8	45.0	−16.7	−14.4	1.4	1.5
	D5	10.4	6.1	15.8	19.4	−6.5	−2.3	1.2	3.4
Size	S1	5.9	2.9	19.1	23.2	−16.0	−9.2	1.0	2.2
	S5	0.3	−0.9	62.2	59.3	−5.5	−2.2	1.0	1.3
IdioVol	IV1	0.6	−0.2	44.4	43.5	−5.4	−6.5	1.0	1.7
	IV5	8.7	5.3	16.5	17.2	−18.7	−9.0	1.1	2.9
Profitability	PR1	9.0	6.0	21.5	23.1	−10.5	−5.4	1.0	3.1
	PR5	2.0	0.8	40.4	34.4	−15.6	−6.9	0.6	1.1

respectively, for the least distressed firms. The results are qualitatively similar for the three other proxies for default options.

Overall, the evidence in this section strongly suggests that the performance of the long-short strategy is driven to a large extent by market mispricing of default options. First, portfolio returns are stronger for stocks that are likely subject to more mispricing, at times when mispricing is likely to be high, and this effect is amplified by the presence of default options. Second, misvalued stocks exhibit characteristics typically associated with over- and undervaluation and this association is much stronger for firms with more important default options.

V. International Evidence

As an out-of-sample exercise, we check the efficacy of our model for a sample of nine developed markets, namely, Australia, Canada, France, Germany, Hong Kong, Italy, Japan, Switzerland, and the United Kingdom. We obtain stock returns and accounting data for international firms from Datastream and Worldscope. The availability of data varies across countries and is scarce before the early 1990s, so we start our sample in 1994. We follow standard filters in cleaning up the data (see, e.g., Ince and Porter (2006) and Griffin, Kelly, and Nardari (2010)). To ensure that we have a reasonable number of stocks for our tests, we also drop country-years with fewer than 100 firms with available data. As we use factor-model alphas in some of our tests, we build factors separately for each country following the approach of Fama and French (1993).

For each country, we first perform valuation on the entire cross section of stocks in the same way as detailed in Section I. We make two adjustments to the procedure to account for data availability issues. First, we estimate the volatility of sales using annual data (based on the past eight years), as quarterly accounting data are sparse for international firms. As for the U.S. sample, in cases where firm-specific volatility cannot be computed, we use the average annual volatility of sales of the firms in the same industry. For robustness, we also report the results using quarterly sales, as, for a large proportion of the sample, the quarterly volatility of sales represents the industry average. Second, we use distance-to-default from Merton's (1974) model instead of CHS (2008) to construct industry distress peer groups and the cost of debt. Merton's model uses only equity value, equity volatility, and the face value of debt, as opposed to CHS and other models that rely on a large set of accounting variables.

Table XIV shows country-specific descriptive statistics. Our summary statistics are consistent with other international studies (see Fama and French (1998) and Fan, Titman, and Twite (2012)), despite some differences in the sample periods. There is substantial variation in the number of firms with available data across countries, from only 175 firms (12,701 firm-months) in Switzerland to 4,305 (551,830 firm-months) in Japan. Market-to-book ratio exhibits two regimes; Hong Kong, Italy, and Japan have fairly low ratios (means of 1.19 to 1.67), while the other seven countries show higher ratios (2.14 to 2.80). Italy and Japan also show relatively high levels of market leverage (mean ratios of 0.40 and 0.36), which is consistent with the low market-to-book ratios in these countries, while for the rest of the countries, mean market leverage ranges between 0.21 and 0.30, which is more comparable to the level of U.S. firms. Finally, monthly stock returns also vary significantly across countries, from an average of 0.12% in Italy to 1.37% in Australia. This suggests that our exercise captures different states of the economy across countries. We view all of these differences in firm characteristics as an advantage because they allow us to examine whether our model can be successful in predicting returns for different types of samples and conditions.

Using the model valuation, we sort stocks into deciles and calculate value- and equal-weighted returns. The returns and alphas of the hedge portfolio that is long undervalued stocks and is short overvalued stocks are reported in Panel A of Table XV. For value-weighted portfolios, the excess returns range from 0.07% for Italy to 1.51% for Hong Kong. For the most part, alphas are similar to excess returns. Focusing on four-factor alphas, we find positive alphas in eight countries, with Italy being an exception, and four of these alphas

Table XIV
Descriptive Statistics by Country

The table presents descriptive statistics separately for nine countries. All variables are winsorized at the 1st and 99th percentiles. Size is market equity value (in millions of dollars). Book-to-market is book equity value divided by market equity value. Monthly stock return is presented in percent. Market leverage is the ratio of total debt book value to the sum of total debt book value and market equity value. For each characteristic, we first calculate the cross-sectional mean and median in each country. We then report the time-series averages of these means/medians. The sample period is 1994 to 2012.

			Size		Market-to-book		Monthly returns		Market leverage	
	# of Firms	# of Firm-Months	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Australia	1,176	64,663	694.5	97.0	2.57	1.56	1.37	0.60	0.21	0.14
Canada	1,456	80,972	1,159.3	178.8	2.14	1.53	0.74	0.30	0.25	0.19
France	1,041	82,902	1,387.0	122.4	2.25	1.57	0.99	0.50	0.30	0.26
Germany	975	78,056	733.0	82.5	2.80	1.74	0.65	0.20	0.26	0.19
Hong Kong	174	19,871	1,901.3	260.9	1.19	0.73	1.33	0.20	0.27	0.22
Italy	280	17,062	1,043.8	215.9	1.67	1.23	0.12	−0.20	0.40	0.38
Japan	4,305	551,830	752.6	134.2	1.38	0.96	0.39	−0.30	0.36	0.34
Switzerland	175	12,701	1,789.1	390.5	2.29	1.55	0.94	0.90	0.24	0.19
United Kingdom	2,059	139,095	962.0	98.8	2.43	1.45	0.63	0.30	0.23	0.17

Table XV

Returns of Portfolios Sorted on Relative Model Value: International Sample

For each country below, each month we sort all stocks into deciles according to the ratio of the equity value implied by our valuation model to actual equity value (decile 1 = most overvalued, decile 10 = most undervalued). The portfolios are value-weighted and held for one month. The table shows the portfolios' mean excess monthly returns (in excess of the risk-free rate) and alphas from factor models. The CAPM one-factor model uses the market factor. The factors in the three-factor model are the Fama and French (1993) factors. The factors in the four-factor model are the Fama and French (1993) factors augmented with a momentum factor. All factors are calculated separately for each country. We also show the equal-weighted (ew) alpha of the long short 10 − 1 portfolio in the last column. All returns and alphas are in percent per month and the corresponding *t*-statistics are in parentheses. Panel A (B) uses annual (quarterly) sales for calculating volatility in the model implementation. The sample period is 1994 to 2012.

	Excess Return	CAPM Alpha	Three-Factor Alpha	Four-Factor Alpha	ew Four-Factor Alpha
Panel A: Volatility Calculated Using Annual Sales					
Australia	1.23	1.29	1.11	1.44	1.99
	(2.15)	(2.22)	(1.89)	(2.34)	(4.60)
Canada	0.87	1.20	1.02	1.19	1.57
	(1.35)	(1.89)	(1.65)	(1.90)	(3.67)
France	0.41	0.46	0.21	0.40	0.95
	(1.02)	(1.12)	(0.54)	(0.98)	(3.57)
Germany	0.92	0.93	0.71	1.10	0.82
	(2.20)	(2.21)	(1.72)	(2.57)	(3.03)
Hong Kong	1.51	1.65	1.58	1.89	1.87
	(2.40)	(2.60)	(2.53)	(2.96)	(3.12)
Italy	0.07	0.02	−0.25	−0.65	0.25
	(0.12)	(0.03)	(−0.44)	(−1.10)	(0.53)
Japan	0.75	0.73	0.38	0.59	0.92
	(1.86)	(1.89)	(1.15)	(1.96)	(4.62)
Switzerland	0.85	0.62	0.42	0.15	1.27
	(1.26)	(0.94)	(0.67)	(0.23)	(2.48)
United Kingdom	0.57	0.53	−0.02	1.03	1.04
	(0.90)	(0.83)	(−0.03)	(1.77)	(3.92)
Panel B: Volatility Calculated Using Quarterly Sales					
Australia	1.71	1.72	1.57	1.94	1.74
	(2.68)	(2.67)	(2.39)	(2.88)	(3.30)
Canada	0.70	1.11	0.94	1.01	1.52
	(1.04)	(1.72)	(1.51)	(1.61)	(3.78)
France	0.34	0.42	0.15	0.27	0.86
	(0.82)	(1.02)	(0.36)	(0.66)	(2.79)
Germany	0.84	0.88	0.47	0.86	0.61
	(1.84)	(1.90)	(1.06)	(1.91)	(2.18)
Hong Kong	2.12	2.29	2.29	2.31	1.14
	(2.65)	(2.83)	(2.92)	(2.82)	(1.80)
Italy	−0.06	−0.20	−0.19	−0.47	0.08
	(−0.10)	(−0.34)	(−0.34)	(−0.83)	(0.19)

(continued)

Table XV—*Continued*

	Excess Return	CAPM Alpha	Three-Factor Alpha	Four-Factor Alpha	ew Four-Factor Alpha
Panel B: Volatility Calculated Using Quarterly Sales					
Japan	1.04	1.03	0.69	0.79	0.96
	(2.84)	(2.88)	(2.25)	(2.62)	(4.87)
Switzerland	1.16	0.98	0.91	0.93	1.28
	(1.99)	(1.70)	(1.61)	(1.57)	(2.34)
United Kingdom	0.69	0.67	0.21	0.88	0.84
	(1.23)	(1.18)	(0.40)	(1.69)	(3.10)

are statistically significant (two more are significant at the 10% significance level). As was the case for the United States, equal-weighted portfolios generate a bigger spread in returns. This suggests that small stocks are typically more mispriced than large stocks in other developed markets as well. With the exception of Italy again, the four-factor alphas are positive and statistically significant for all other eight countries; the four-factor alpha is highest at 1.99% for Australia and lowest at 0.82% for Germany, with t-statistics of 2.48 to 4.62. Panel B shows fairly similar results when using quarterly data in calculating the volatility of sales; the means of the four-factor alphas on the value- and equal-weighted hedge portfolios are 0.95% and 1.00% per month, compared to 0.79% and 1.19% using annual data.

We show the results of the Fama and MacBeth (1973) regressions in Table XVI. We run regressions separately for each country. The dependent variable is the monthly excess stock return. The independent variables are log market capitalization (SZ), log market-to-book (MB), past six-month return ($R6$), and relative model value ($MODEL$). The coefficients on size, market-to-book, and past returns are mostly consistent with extant evidence. The coefficient of interest to us is that on relative model value. Since the magnitudes of the coefficients are difficult to compare across countries, we focus on the statistical significance. The coefficient is positive for all nine countries we analyze and significant for six of these countries (t-statistics from 2.32 to 3.18). The coefficient is also significant at the 10% level for Switzerland (t-statistic of 1.82).

To conclude, the evidence from the sample of developed countries is consistent with that for the United States, providing further evidence that our model is able to identify stocks that are mispriced.

VI. Conclusion

Equities are embedded with an option to default. We believe that a meaningful equity valuation model should take this option into account. An important question is whether investors recognize this insight and account for the default option when valuing equity. To address this question, we build a model

Table XVI

Fama-MacBeth Regressions on Relative Model Value: International Sample

We run cross-sectional Fama and MacBeth (1973) regressions each month of excess stock returns separately for each country listed below. The independent variables are log market capitalization (*SZ*), log market-to-book (*MB*), past six-month return (*R*6), and relative model value (*MODEL*). *MB* is calculated as the ratio of current market value divided by book value in the previous quarter. We skip one month in calculating *R*6. *MODEL* is the log of the ratio of the equity value implied by our valuation model to actual equity value. All coefficients are multiplied by 100 and Newey and West (1987) corrected *t*-statistics (with six lags) are in parentheses. The sample period is 1994 to 2012.

	Australia	Canada	France	Germany	Hong Kong	Italy	Japan	Switzerland	United Kingdom
CNST	1.075	0.668	0.410	0.565	0.943	−0.157	−0.383	0.365	0.952
	(1.28)	(0.96)	(1.01)	(0.76)	(1.00)	(−0.27)	(−0.35)	(0.58)	(1.15)
SZ	0.129	−0.012	−0.021	0.067	−0.015	0.101	0.018	−0.011	−0.109
	(0.49)	(−0.13)	(−0.42)	(0.64)	(−0.14)	(1.53)	(0.08)	(−0.13)	(−1.02)
MB	−0.234	−0.019	0.041	−0.140	−0.005	−0.506	−0.254	0.171	0.072
	(−1.75)	(−0.20)	(0.31)	(−1.80)	(−0.02)	(−3.27)	(−3.34)	(0.79)	(0.88)
Ret6	2.942	1.439	1.015	1.048	−0.114	1.755	−0.325	2.177	1.633
	(3.81)	(4.28)	(2.24)	(2.59)	(−0.13)	(2.66)	(−0.60)	(2.37)	(3.84)
MODEL	0.201	0.187	0.257	0.148	0.100	0.035	0.142	0.198	0.125
	(2.78)	(2.32)	(3.18)	(3.11)	(0.85)	(0.41)	(2.50)	(1.82)	(2.39)

that accounts for the value of the option to default or abandon the firm. Our model is capable of separating over- and undervalued stocks. The long-short strategy that buys stocks that are classified as undervalued by our model and shorts stocks classified as overvalued generates an annualized four-factor alpha of about 11%. This performance is robust to various sample splits and holding periods. Furthermore, a similar investment strategy produces significantly higher returns for stocks with a relatively high value of the default option, namely, distressed, highly volatile, and low profitability stocks, which highlights the importance of the option to default as the key ingredient of our model. International evidence from nine of the largest developed markets supports our U.S.-based results. This suggests that default options are mispriced by the market and, more generally, investors do not fully recognize the option-like nature of equities and hence do not value them accordingly.

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Appendix A: Numerical Details on the Valuation Model

The first step is to find a value of the firm that survives until year 5 and pays off its long-term debt. We assume that, at the end of year 5, the firm refinances by issuing perpetual-coupon debt in an amount to match the average two-digit SIC market leverage ratio. Thus, we assume that, over time period dt, debt holders receive a payment of $c_i dt$. This assumption allows for analytical solutions for security values and substantially speeds up our valuation procedure. (Note that, in the first five years, we assume that coupon payments are annual and given by I_{it}.) We assume refinancing to average industry leverage, as opposed to inferring the optimal leverage from the model, due to the known tendency of structural contingent-claim models to predict optimal leverage ratios that appear too high compared with their empirical counterparts.

The net instantaneous post-refinancing cash flow to equity holders is

$$\begin{aligned} CF_{it} = & \left[(1-\tau)(x_{it} - c_i - F_i) + \tau Dep_{it} - Capex_{it}\right] \\ & \times \left[1 + \eta \mathbf{1}_{(1-\tau)(x_{it}-c_i-F_i)+\tau Dep_{it} - Capex_{it} < 0}\right] dt, \end{aligned} \tag{A1}$$

where the coupon amount is c_i. The cash flow to bondholders is $c_i dt$. Note that the additional cost of financial distress η is incurred if $x_{it} < x^*$, where

$$(x^* - c_i - F_i)(1-\tau) + \tau Dep_{it} - Capex_{it} = 0.$$

Because we assume that the gross margin ratio, GM_{it}, as well as the depreciation-to-sales and capex-to-sales ratios, stay constant over time, x^* is given by

$$x^* = \frac{(c_i + F_i)(1-\tau)}{1 - \tau + \left(\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}\right)/GM_{it}}. \tag{A2}$$

The cash flows to equity holders, and hence the value of equity, depend on whether the current value of x_{it} is above or below the threshold x^*. The cash flows in equation (A1) above can be rewritten as

$$CF_{it} = \Bigg[x_{it} \left(\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right) \times \left(1 + \eta \mathbf{1}_{x_{it}\left(\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right) - (c_i + F_i)(1-\tau) < 0} \right) - (c_i + F_i)(1-\tau) \Bigg] dt. \quad \text{(A3)}$$

Standard arguments show that the value of equity can thus be given by

$$E(x_{it}) = \begin{cases} Ax_{it}^{\beta_1} + Bx_{it}^{\beta_2} + \left[\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right] \frac{x_{it}}{r-\mu} - (1-\tau)\frac{c_i + F_i}{r} & \text{if } x_{it} \geq x^* \\ Cx_{it}^{\beta_1} + Dx_{it}^{\beta_2} + (1+\eta) \left\{ \left[\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right] \frac{x_{it}}{r-\mu} - (1-\tau)\frac{c_i + F_i}{r} \right\} & \text{otherwise,} \end{cases} \quad \text{(A4)}$$

where β_1 and β_2 are the positive and negative root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \mu_Q\beta - r = 0$, and A, B, C, and D are constants. Equation (A4) must be solved subject to the following boundary conditions:

$$A = 0,$$

$$Bx^{*\beta_2} + \left[\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right] \frac{x^*}{r-\mu_Q} - (1-\tau)\frac{c_i + F_i}{r} = Cx^{*\beta_1} + Dx^{*\beta_2} + (1+\eta) \left\{ \left[\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right] \frac{x^*}{r-\mu_Q} - (1-\tau)\frac{c_i + F_i}{r} \right\},$$

$$\beta_2 Bx^{*\beta_2-1} + \left[\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right] \frac{1}{r-\mu_Q} = \beta_1 Cx^{*\beta_1-1} + \beta_2 Dx^{*\beta_2-1} + (1+\eta) \left\{ \frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right\} \frac{1}{r-\mu_Q},$$

$$Cx_d^{\beta_1} + Dx_d^{\beta_2} + (1+\eta) \left\{ \left[\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right] \frac{x_d}{r-\mu_Q} \right.$$

$$- (1-\tau)\frac{c_i + F_i}{r}\Bigg\} = 0,$$

$$\beta_1 C x_d^{\beta_1 - 1} + \beta_2 D x_d^{\beta_2 - 1} + (1+\eta) \left[\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{1}{r-\mu_Q} = 0. \tag{A5}$$

The first boundary condition precludes bubbles as x increases, the second and third conditions ensure that the value functions and their first derivatives match at x^*, and the fourth and fifth conditions are the value-matching and smooth-pasting conditions that ensure optimality of the default threshold x_d. Together, these conditions comprise a system of four nonlinear equations with four unknowns (B, C, D, and x_d) that must be solved numerically. By solving this system, we find the post-refinancing value of equity in year 5, $E(x_{i5})$.

The value of debt is given by

$$D(x_{i5}) = \frac{c_i}{r} + \left(\frac{x_{i5}}{x_d}\right)^{\beta_2}\left[(1-\alpha)\left(V_U(x_d) - \frac{c_i}{r}\right)\right], \tag{A6}$$

where $V_U(x_d)$ is the value of the unlevered firm and α is the cost of bankruptcy (upon default, debt holders get this unlevered value net of bankruptcy costs). Note that $V_U(x_d)$ is always positive because the value of equity is decreasing in the total fixed cash outflow $c_i + F_i$, and so is the optimal default/exit boundary. This implies that, for $c_i > 0$, the optimal default threshold x_d is greater than the optimal exit threshold of the same firm with no debt and hence the value of that firm at x_d is positive. When implementing this procedure, we set $\alpha = \eta = 15\%$.

For a given x_{i5} (we assume that long-term debt is repaid in five years), we find the value of c_i such that $\frac{D(x_{i5})}{E(x_{i5})+D(x_{i5})}$ is equal to the average two-digit SIC leverage ratio in the last three years. If we are unable to find this solution (e.g., for high enough values of fixed costs), we assume that the firm remains unlevered over the rest of its life. The pre-refinancing equity value (after repayment of the initial debt) equals the sum of the post-refinancing value and the proceeds from issuing debt:

$$E'(x_{i5}) = E(x_{i5}) + D(x_{i5}). \tag{A7}$$

Once we find the terminal value of equity in year 5, $E'(x_{i5})$, we solve the model numerically and compute the optimal default boundary and equity values for all $t \leq T = 5$. For that purpose, we introduce the variable $y_t = \log(x_t)$ that follows an arithmetic Brownian motion under the risk-neutral measure:

$$dy_t = \left(\mu_Q - \frac{\sigma^2}{2}\right)dt + \sigma dW_t. \tag{A8}$$

We then discretize the problem by using a two-dimensional grid $N_y \times N_t$ with the corresponding increments of y and t given by dy and dt, where $dy = (y_{\max} - y_{\min})/N_y$ and $dt = T/N_t$, where $T = 5$. To get a reasonable balance between execution speed and accuracy, we set $dt = 0.1$, $y_{\min} = -5$, and $y_{\max} = 10$ when implementing this algorithm.

We iterate valuations backward using a binomial approximation of the Brownian motion (see, e.g., Dixit and Pindyck (1994)). At each node, equity holders have an option to default. They will default if the present value (under Q) of running the firm for one more period is negative:

$$\begin{aligned} E(ndy, mdt) = \max \Bigg\{ & e^{-rdt}[p_u E((n+1)dy, (m+1)dt) + p_d E((n-1)dy, (m+1)dt)] \\ & + \left[e^{n\times dy} \left(\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau) \right) \right. \\ & \times \left(1 + \eta \mathbf{1}_{x_{it}\left(\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right) - (I_{it}+F_i)(1-\tau) < 0} \right) \\ & \left. - (I_{it} + F_i)(1-\tau) \right] dt - D_{it}, 0 \Bigg\}, \end{aligned} \tag{A9}$$

where

$$p_u = 0.5 + \left(\mu_Q - \frac{\sigma^2}{2} \right) \frac{\sqrt{dt}}{2\sigma}, \quad p_d = 1 - p_u, \quad \text{and} \quad dy = \sigma \sqrt{dt}.$$

Equation (A9) shows that, at each node, the value of equity is given by the discounted present value of equity in the next time period plus the cash flows that equity holders receive over the period dt. If this value is negative, then the firm is below the optimal default boundary and it is optimal for equity holders to default, in which case the value of equity is zero. (We assume that the APR is enforced if bankruptcy occurs and the residual payout to equity holders is zero).

Appendix B: Distress Measure

CHS (2008) use logit regressions to predict failure probabilities. We use their model for predicting bankruptcy over the next year (model with lag 12 in their Table IV) as our baseline model. This model, which is repeated below, gives the probability of bankruptcy/failure from a logit model as

$$\begin{aligned} CHS_t = & -9.16 - 20.26\, NIMTAAVG_t + 1.42\, TLMTA_t - 7.13\, EXRETAVG_t \\ & + 1.41\, SIGMA_t - 0.045\, RSIZE_t - 2.13\, CASHMTA_t + 0.075\, MB_t \\ & - 0.058\, PRICE_t, \end{aligned} \tag{B1}$$

where $NIMTA$ is the net income divided by the market value of total assets (the sum of market value of equity and book value of total liabilities), $TLMTA$ is the book value of total liabilities divided by the market value of total assets, *EXRET* is the log of the ratio of the gross returns on the firm's stock and on the S&P500 index, *SIGMA* is the standard deviation of the firm's daily stock return over the past three months, *RSIZE* is ratio of the log of the firm's equity market capitalization to that of the S&P500 index, *CASHMTA* is the ratio of the firm's cash and short-term investments to the market value of total assets, *MB* is the market-to-book ratio of the firm's equity, and *PRICE* is the log price per share. *NIMTAAVG* and *EXRETAVG* are moving averages of *NIMTA* and *EXRET*, respectively, and constructed as (with $\phi = 2^{-\frac{1}{3}}$)

$$NIMTAAVG_{t-1,t-12} = \frac{1-\phi^3}{1-\phi^{12}}\left(NIMTA_{t-1,t-3} + \cdots + \phi^9 NIMTA_{t-10,t-12}\right),$$
$$EXRETAVG_{t-1,t-12} = \frac{1-\phi}{1-\phi^{12}}\left(EXRET_{t-1} + \cdots + \phi^{11} EXRET_{t-12}\right). \tag{B2}$$

All accounting data are taken with a lag of three months for quarterly data and a lag of six months for annual data. All market data used in calculating the distress measure of equation (B1) are the most current data. We winsorize all inputs at the 5^{th} and 95^{th} percentiles of their pooled distributions across all firm-months (winsorizing at the 2^{nd} and 98^{th} percentiles has no material impact on our results), and *PRICE* is truncated above at \$15. Further details on the data construction are provided by CHS (2008); we refer the interested reader to their paper.[22]

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[22] There are two minor differences between CHS's (2008) approach and ours. First, CHS eliminate stocks with fewer than five nonzero daily observations during the last three months, and then replace missing *SIGMA* observations with the cross-sectional mean *SIGMA* in estimating their bankruptcy prediction regressions. We do not make this adjustment. Second, CHS treat firms that fail as equivalent to delisted firms, even if CRSP continues to report returns for these firms. We do not make this adjustment either.

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Appendix S1: Internet Appendix.
Replication Code.